Macro II/Aussenwirtschaft
Lecture Slides No 3

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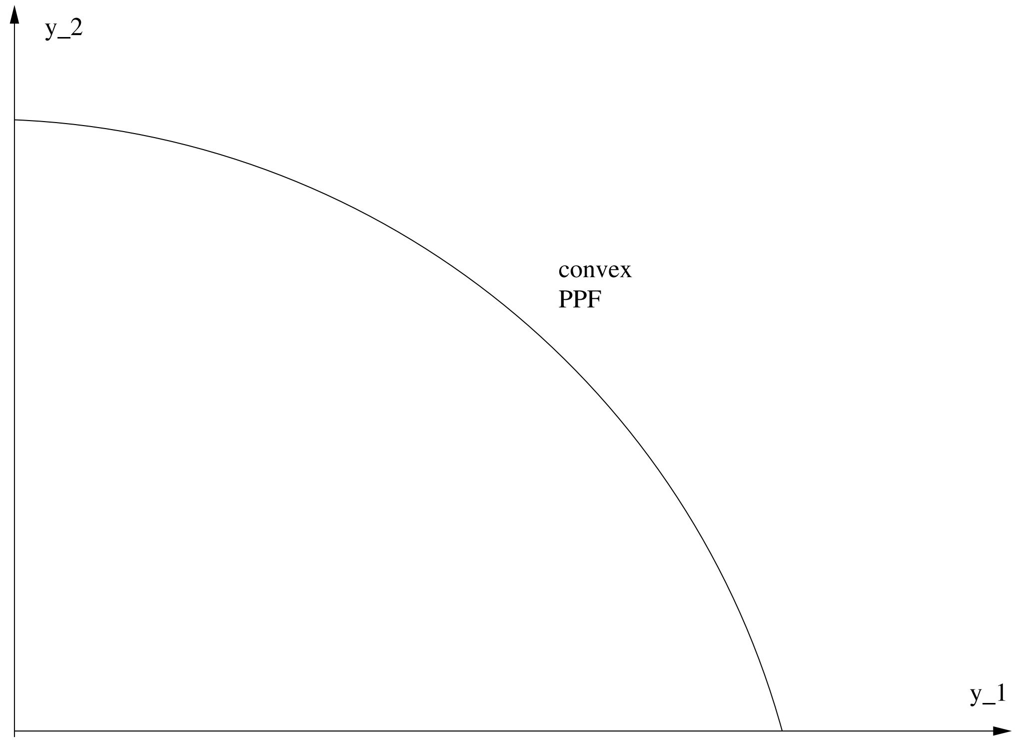
Heckscher-Ohlin - neoclassical model

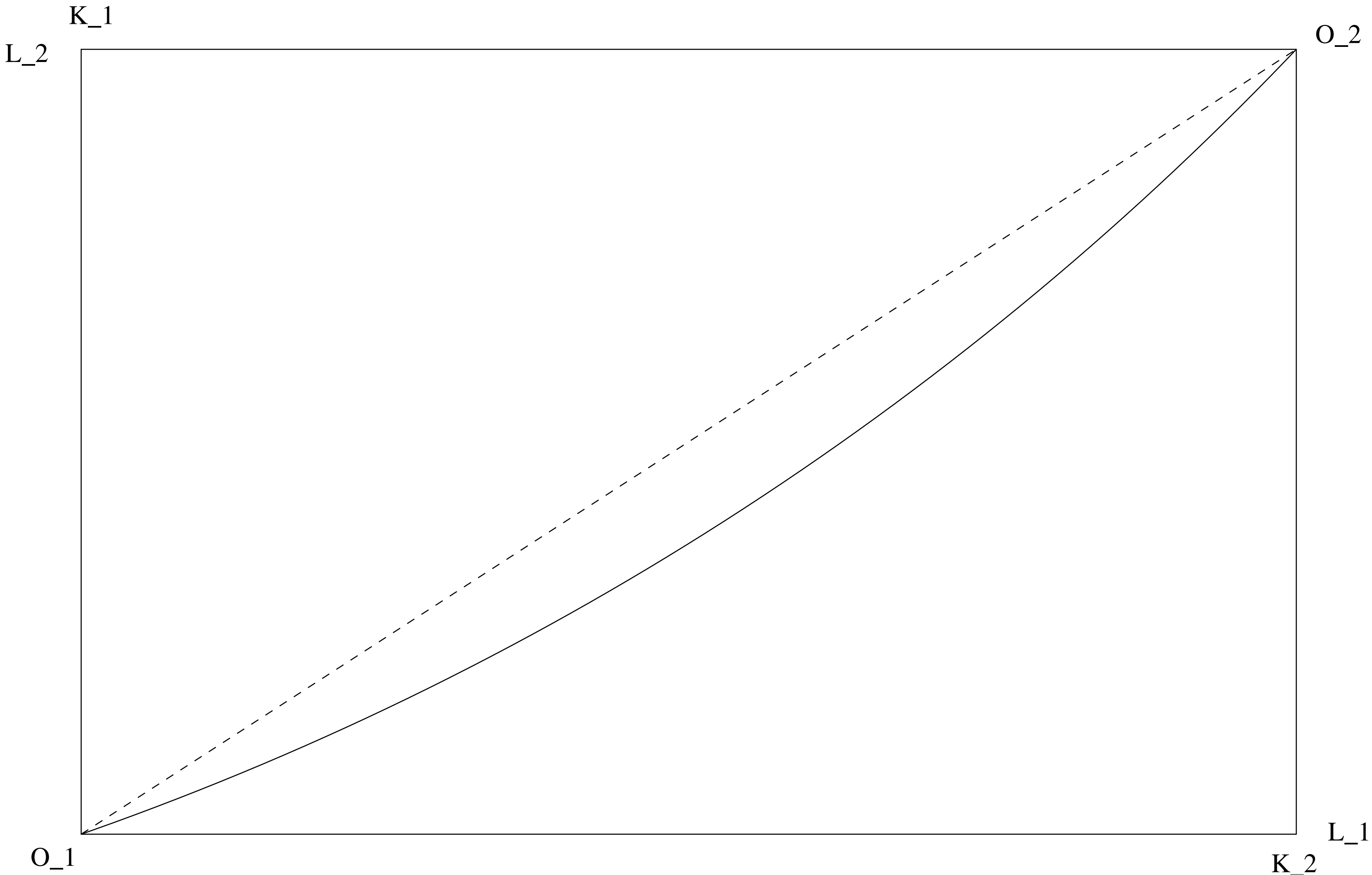
- Eli Heckscher (1879–1952) and Bertil Ohlin (1899–1979)
- latter a student of the former, received Nobel in 1977
- explain trade as driven by differences in factor endowments
- 2x2x2 — 2 factors, 2 sectors, 2 countries
- factors traditionally labor and capital
- more modern interpretation: unskilled and skilled labor, or human capital

Formal setup

- 2 factors: K and L   2 sectors: 1 and 2
- sector 1 assumed to be labor intensive
- sector 2 assumed to be capital intensive
- production functions: $y_i = f_i(L_i, K_i)$
- increasing, concave, homogeneous of degree 1, i.e. CRS
- factors immobile internationally, perfectly mobile between sectors
- resource constraints: $L_1 + L_2 \leq L$ and $K_1 + K_2 \leq K$

$\max y_2 = f_2(L_2, K_2)$ subject to $y_1 = f_1(L_1, K_1)$ and 2 resource constraints gives the convex PPF $y_2 = h(y_1; L, K)$




K_1
L_2
O_2
L_1
O_1
K_2

GDP function

- $G(p_1, p_2, L, K) = \max_{y_1,y_2} p_1 y_1 + p_2 y_2$ s.t. $y_2 = h(y_1; L, K)$
- substitute in the constraint and differentiate w.r.t. y_1
- $p_1 + p_2 \frac{\partial h}{\partial y_1} = 0$
- $\frac{p_1}{p_2} = -\frac{y_2}{y_1}$
- in words: relative price determines production point on PPF
- where does the price come from?
- interplay of supply and demand

GDP fct continued

- for small country: world market price is given
- convenient property of GDP function
- $\frac{\partial G}{\partial p_i} = y_i + (p_1 \partial y_1 / \partial p_i + p_2 \partial y_2 / \partial p_i)$
- use envelope theorem to make term in parentheses disappear
- difficult to define equilibrium in quantities
- better to do it in prices, i.e. use dual approach

Consider unit cost functions

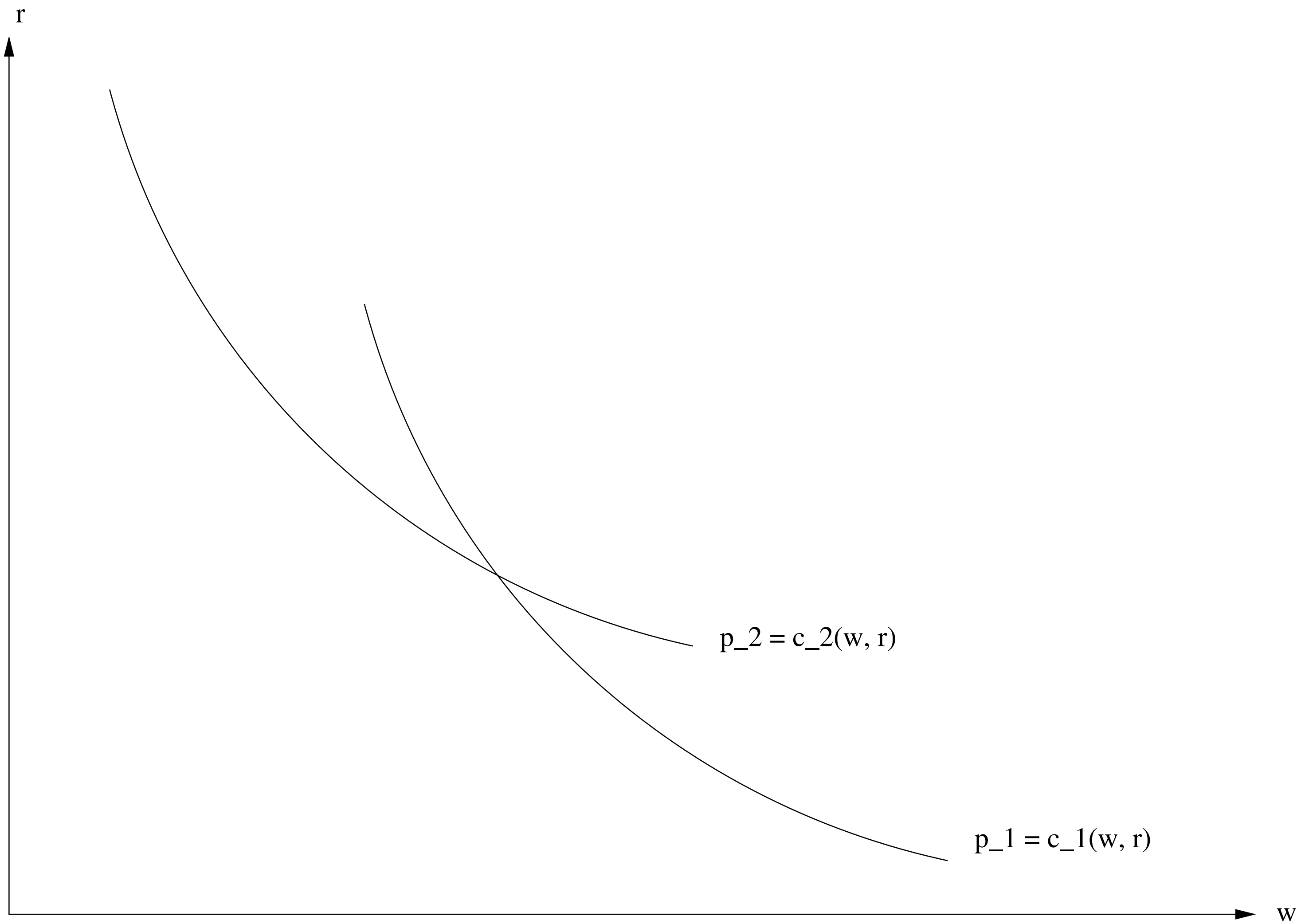
- $c_i(w,r) = \min_{L_i,K_i}(wL_i + rK_i | f_i(L_i,K_i) \geq 1)$
- differentiate w.r.t. factor price and use envelope
- $\frac{\partial c_i}{\partial w} = a_{iL} + (w\partial a_{iL}/\partial w + r\partial a_{iK}/\partial w)$
- to see why the term in parentheses is zero (envelope)
- totally differentiate constraint $f_i(a_{iL}, a_{iK}) = 1$ w.r.t. w

Equilibrium conditions

- zero profit in both sectors because of CRS
- $p_1 = c_1(w, r)$ and $p_2 = c_2(w, r)$
- as well as resource constraints (with equality)
- $a_{1L}y_1 + a_{2L}y_2 = L$ and $a_{1K}y_1 + a_{2K}y_2 = K$
- 4 equations in 4 unknowns: w, r, y_1, and y_2
- note recursive structure: 1st two determine (w, r) (assuming no reversals)

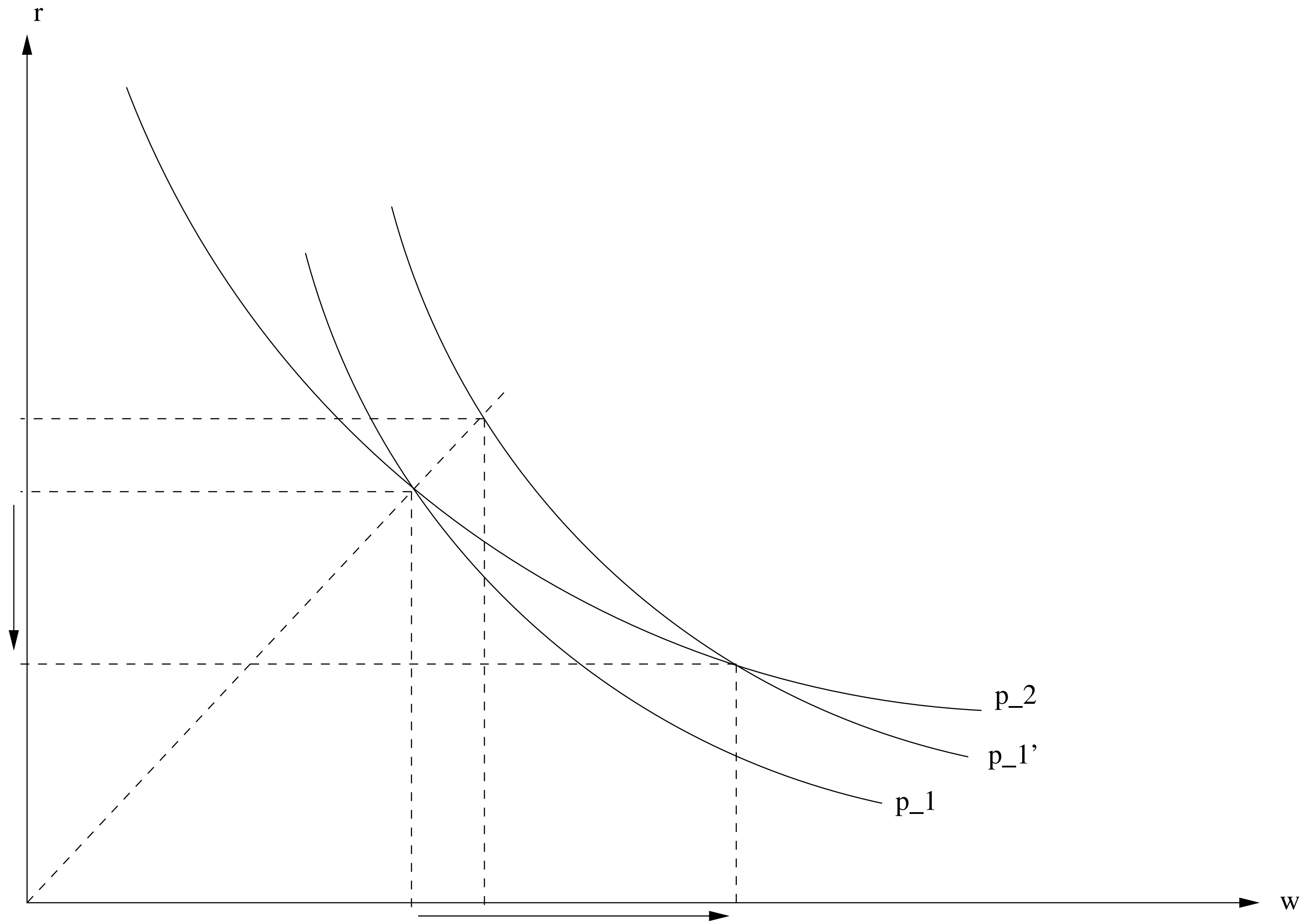
First result: FPE (factor price equalization)

- first two equations (zero profit) determine (w, r)
- 2 eqtns are the same for both countries
- b/c technology and hence cost fcts are the same
- world market output prices p_i are the same for all countries
- hence resulting factor prices must be equal across both countries
- Samuelson derived this result, and admitted it's not necessarily realistic
- but it follows from assumptions, in particular same tech


r
w
p_2 = c_2(w, r)
p_1 = c_1(w, r)

Second result: Stolper–Samuelson

- how do factor prices change in response to output price change
- suppose relative output price changes in the world market
- how does this affect the wage w and interest rate r
- important as it determines whether factor owners support or oppose trade liberalization
- consider percentage change $\hat{x} = dx/x$
- goes by the name Jones' hat algebra due to Ron Jones
- look at this graphically, then mathematically



r
p_2
p_1'
p_1
w

Stolper–Samuelson continued

- consider increase in relative output price $p = p_1/p_2$
- to be concrete: suppose p_1 increase, p_2 doesn't change
- we see from the diagram that w increases and r decreases
- more then that: w increases by more than p_1
- r decreases more than p_2 (which hasn't budged)
- we get the so-called magnification effect
- $\hat{w} > \hat{p_1} > \hat{p_2} > \hat{r}$ (or $\hat{w} < \hat{p_1} < \hat{p_2} < \hat{r}$ if p_1/p_2 decreases)

Stolper–Samuelson continued

- relative price of factor used intensively in production of product whose relative price has increased increases by more
- relative price of factor used intensively in production of product whose relative price has decreased decreases by more
- magnification implies rel factor price moves by more than rel output price
- implication for real factor rewards $w/price$ and $r/price$
- one goes up, the other down
- one factor gains from trade, the other loses

Mathematically — hat algebra

$$\begin{aligned} p_1 &= a_{1L}w + a_{1K}r \\ p_2 &= a_{2L}w + a_{2K}r \end{aligned}$$

$$\begin{aligned} dp_1 &= a_{1L}dw + a_{1K}dr \\ dp_2 &= a_{2L}dw + a_{2K}dr \end{aligned}$$

$$\begin{aligned} \frac{dp_1}{p_1} &= \frac{wa_{1L}}{p_1}\frac{dw}{w} + \frac{ra_{1K}}{p_1}\frac{dr}{r} \\ \frac{dp_2}{p_2} &= \frac{wa_{2L}}{p_2}\frac{dw}{w} + \frac{ra_{2K}}{p_2}\frac{dr}{r} \end{aligned}$$

$$
\begin{aligned}
\hat{p}_1 &= \theta_{1L}\hat{w} + \theta_{1K}\hat{r} \\
\hat{p}_2 &= \theta_{2L}\hat{w} + \theta_{2K}\hat{r}
\end{aligned}
$$

$$
\begin{pmatrix} \hat{p}_1 \\ \hat{p}_2 \end{pmatrix} = \begin{pmatrix} \theta_{1L} & \theta_{1K} \\ \theta_{2L} & \theta_{2K} \end{pmatrix} \begin{pmatrix} \hat{w} \\ \hat{r} \end{pmatrix}
$$

$$
\begin{pmatrix} \hat{w} \\ \hat{r} \end{pmatrix} = \begin{pmatrix} \theta_{1L} & \theta_{1K} \\ \theta_{2L} & \theta_{2K} \end{pmatrix}^{-1} \begin{pmatrix} \hat{p}_1 \\ \hat{p}_2 \end{pmatrix}
$$

$$\begin{pmatrix} \hat{w} \\ \hat{r} \end{pmatrix} = \frac{1}{|\Theta|} \begin{pmatrix} \theta_{2K} & -\theta_{1K} \\ -\theta_{2L} & \theta_{1L} \end{pmatrix} \begin{pmatrix} \hat{p}_1 \\ \hat{p}_2 \end{pmatrix}$$

determinant:

$$\begin{aligned} |\Theta| &= \theta_{1L}\theta_{2K} - \theta_{1K}\theta_{2L} \\ &= \theta_{1L}(1-\theta_{2L}) - (1-\theta_{1L})\theta_{2L} \\ &= \theta_{1L} - \theta_{2L} = \theta_{2K} - \theta_{1K} > 0 \end{aligned}$$

$$
\begin{aligned}
\hat{w} &= (\theta_{2K}\hat{p}_1 - \theta_{1K}\hat{p}_2)/|\Theta| \\
&= \frac{(\theta_{2K} - \theta_{1K})\hat{p}_1 + \theta_{1K}(\hat{p}_1 - \hat{p}_2)}{\theta_{2K} - \theta_{1K}} \\
&> \hat{p}_1 \qquad \text{if} \qquad \hat{p}_1 > \hat{p}_2
\end{aligned}
$$

$$
\begin{aligned}
\hat{r} &= (\theta_{1L}\hat{p}_2 - \theta_{2L}\hat{p}_1)/|\Theta| \\
&= \frac{(\theta_{1L} - \theta_{2L})\hat{p}_2 - \theta_{2L}(\hat{p}_1 - \hat{p}_2)}{\theta_{1L} - \theta_{2L}} \\
&< \hat{p}_2 \qquad \text{if} \qquad \hat{p}_1 > \hat{p}_2
\end{aligned}
$$

Note: inequalities reverse if $\hat{p}_1 < \hat{p}_2$