Economies of Scope and Trade

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Overview

Literature overview

Product symmetry	Product asymmetry
Products symmetric on both the demand and supply side	*Products asymmetric on the demand side*
Allanson and Montagna (IJIO 2005)	Bernard, Redding and Schott (AER 2010, QJE 2011)
Nocke and Yeaple (IER 2014)	
	Products asymmetric on the supply (cost) side
	Arkolakis, Ganapati and Muendler (2015)
	Mayer, Melitz and Ottaviano (AER 2014)
Cannibalization	
Ju (RIE 2003)	Eckel and Neary (RES 2010)
Feenstra and Ma (2008)	
Baldwin and Gu (2009)	Eckel, Iacovone, Javorcik and Neary (JIE 2015)
Dhingra (AER 2013)	Qiu and Zhou (JIE 2013)

Overview - Preview of results
Scope adjustment to trade liberalization

classes of scope reactions:

- economy-wide reduction
  - Baldwin and Gu (2009)
  - Eckel and Neary (2010)
  - Mayer, Melitz and Ottaviano (2014)
- ambiguous reaction
  - Bernard, Redding and Schott (2011)
- heterogeneous reaction throughout the firm distribution
  - Dhingra (2013)
  - Qiu and Zhou (2013)
  - Nocke and Yeaple (2014)

Eckel, Carsten and Peter Neary (2010). Multi-Product Firms and Flexible Manufacturing in the Global Economy, *Review of Economic Studies* 77(1), pp. 188-217.

Preferences and Demand

two-tier utility function:

$$U[u(z)] = \int_0^1 u(z)dz \tag{1}$$

with

$$u(z) = a\int_0^N q(i)di - \frac{1}{2}b\left[(1-e)\int_0^N q(i)^2 di + e\left\{\int_0^N q(i)di\right\}^2\right]$$

$q(i)$: consumption of (horizontally diff.) product variety i, $i \in [0, N]$ and
N: measure of diff. varieties produced in each industry z, $z \in [0, 1]$

Eckel and Neary (2010)

utility maximization problem:

$$\max_{q(i)} \quad U[u(z)] \quad \text{subject to}$$

$$\int_0^1 \int_0^N p(i)q(i)didz \leq I$$

$p(i)$: price of variety i and I: individual income

FOC: inverse individual demand function:

$$\lambda p(i) = a - b\left[(1-e)q(i) + e\int_0^N q(i)di\right] \qquad (2)$$

λ: Lagrange multiplier (consumer's marginal utility of income)

L (homogeneous) consumers in each of k identical countries, integrated goods markets and free trade (single variety price worldwide)

Eckel and Neary (2010)

market demand for variety i: $x(i) = kLq(i)$

inverse world market demand function:

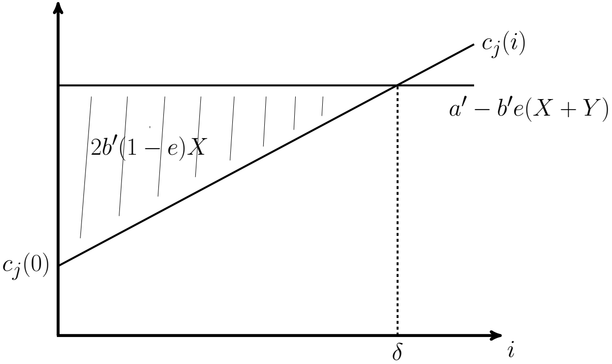
$$p(i) = a' - b'[(1-e)x(i) + eY] \tag{3}$$

$a' \equiv a/\lambda$, $b' \equiv b/\lambda kL$ and $Y \equiv \int_0^N x(i)di$: industry output

Production and Supply
"flexible manufacturing" technology (core competence):
$c_j(i)$: marginal cost of firm j to produce variety i (independent of output, but different across products: $c_j' > 0$ and $c_j(0) = c_j^0$; e.g. linear: $c_j(i) = c_j^0 + \gamma i$)

Eckel and Neary (2010)

Figure 1



Eckel and Neary (2010)

single-stage Cournot game

profit maximization problem:

$$\max_{x_j(i)} \quad \pi_j = \int_0^{\delta_j} [p_j(i) - c_j(i)]\, x_j(i) di - F$$

δ_j: mass of products produced (scope) and F: fixed cost

FOC: (i) scale

$$\frac{\partial \pi_j}{\partial x_j(i)} = p_j(i) - c_j(i) - b' \left[(1-e)x_j(i) + eX_j\right] = 0 \qquad (4)$$

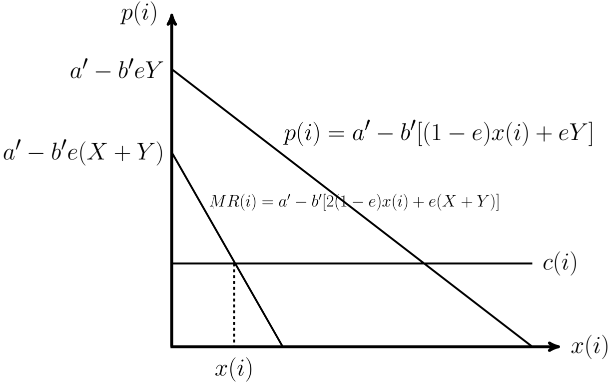
▸ proof

$X_j \equiv \int_0^{\delta_j} x_j(i)di$: firm's aggregate output

$$x_j(i) = \frac{a' - c_j(i) - b'e\,(X_j + Y)}{2b'(1-e)} \qquad (5)$$

Eckel and Neary (2010)

Figure 2



Eckel and Neary (2010)

$$p_j(i) = \frac{1}{2}\left[a' + c_j(i) - b'e(Y - X_j)\right] \tag{6}$$

(ii) scope

$$\frac{\partial \pi_j}{\partial \delta_j} = [p_j(\delta_j) - c_j(\delta_j)]\, x_j(\delta_j) = 0 \tag{7}$$

product range: output of the marginal variety (δ_j) zero: $x_j(\delta_j) = 0$

$$c_j(\delta_j) = a' - b'e\,(X_j + Y) \tag{8}$$

$$p_j(\delta_j) = a' - b'eY$$

labour productivity (LP) of multi-product firms:

labour as the only factor of production and economy-wide and perfectly competitive labour market

unit cost of producing each variety:

$$c(i) = w\gamma(i)$$

Eckel and Neary (2010)

total labour input:

$$l = \int_0^\delta \gamma(i)x(i)di$$

$$\frac{d \ln \text{LP}}{d \ln \theta} = \frac{\int_0^\delta h(i) \frac{d\, x(i)}{d \ln \theta} di}{\int_0^\delta h(i)x(i)di} - \frac{d \ln l}{d \ln \theta} \tag{9}$$

θ: any exogenous variable and $h(i)$: weight of variety i

$$x(i) = \frac{w\left[\gamma(\delta) - \gamma(i)\right]}{2b'(1-e)}$$

$$l = \frac{w\beta(\delta)}{2b'(1-e)} \quad \text{with} \quad \beta(\delta) \equiv \int_0^\delta \gamma(i)\left[\gamma(\delta) - \gamma(i)\right] di$$

$$\frac{d \ln \text{LP}}{d \ln \theta} = \frac{\partial \ln \text{LP}}{\partial \ln \delta} \frac{d \ln \delta}{d \ln \theta}$$

proof

Eckel and Neary (2010)

choice of weights $h(i)$:

$$\left.\frac{\partial \ln \mathrm{LP}}{\partial \ln \delta}\right|_{h(i)=\gamma(i)} = \frac{\int_0^\delta \gamma(i)\frac{\partial x(i)}{\partial \ln \delta}di}{\int_0^\delta \gamma(i)x(i)di} - \frac{\partial \ln l}{\partial \ln \delta} = 0$$ proof

Proposition 1: With given technology, any shock which raises the product range δ
(a) leaves LP unchanged when output changes are marginal cost-weighted,
(b) reduces LP when output is a simple aggregate proof and
(c) reduces LP but by less when output changes are price-weighted proof.

Eckel and Neary (2010)

Industry Equilibrium

symmetric Cournot oligopoly with an exogenously given number of firms m in each of k countries

industry output: $Y = kmX$

FOC for scope (rewrite (8)):

$$w\gamma(\delta) = a' - e(1+km)b'X \;\Rightarrow\; \text{scope: } \delta(X)$$

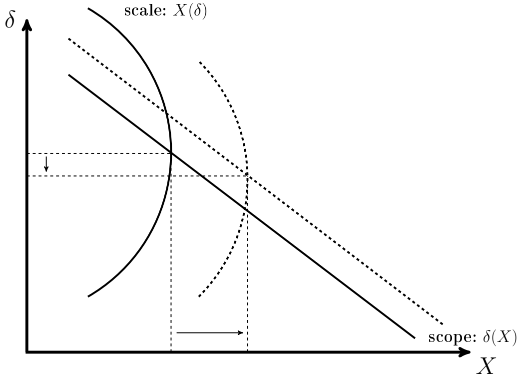
FOC for scale (integrate over (5)):

$$X = \frac{\left(a' - w\mu'_\gamma\right)\delta}{\triangle_1 \; b'} \text{ with } \triangle_1 \equiv 2(1-e)+e\delta(1+km) > 0 \text{ proof } \Rightarrow \text{ scale: } X(\delta)$$

with $\mu'_\gamma \equiv \frac{1}{\delta}\int_0^\delta \gamma(i)di$

$$\frac{d\ln X}{d\ln\delta} = \frac{a' - w\gamma(\delta) - e(1+km)b'X}{a' - w\mu'_\gamma}$$ proof

Eckel and Neary (2010)
Figure 3



Eckel and Neary (2010)

Effects of Globalization

globalization: increase in the number of countries k participating in the global economy

two channels:

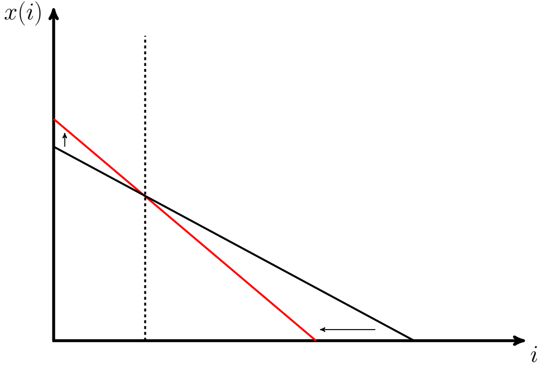
- market-size effect ($L \uparrow$)
- competition effect ($m \uparrow$)

Proposition 2: The *market-size effect* of an increase in k is an equi-proportionate increase in the output of each variety and of total output, but no change in firm scope.

Proposition 3: The *competition effect* of an increase in k is a uniform absolute fall in the output of each variety, coupled with falls in both total firm output and firm scope, but a rise in industry output.

Eckel and Neary (2010)

Figure 4



Eckel and Neary (2010)

full effect:

- on firm output:

$$\frac{d\ln X}{d\ln k} = 1 - \frac{e\delta km}{\triangle_1} \quad \text{proof} \tag{10}$$

where $\triangle_1 - e\delta km = \triangle_0 \ (\equiv 2(1-e) + e\delta) > 0$

- on variety output:

$$\frac{d\ln x(i)}{d\ln k} = 1 - \frac{ekm\alpha(\delta)}{\triangle_1\left[\gamma(\delta) - \gamma(i)\right]} = \frac{\triangle_0}{\triangle_1} + \left(1 - \frac{\triangle_0}{\triangle_1}\right)\frac{\mu'_\gamma - \gamma(i)}{\gamma(\delta) - \gamma(i)} \quad \text{proof} \tag{11}$$

$\widetilde{\gamma}^{PE}$: labour requirement of the threshold variety whose output is unchanged

$$\widetilde{\gamma}^{PE} = \frac{\triangle_0}{\triangle_1}\gamma(\delta) + \left(1 - \frac{\triangle_0}{\triangle_1}\right)\mu'_\gamma \quad \text{proof}$$

Eckel and Neary (2010)

Proposition 4: The *total effect* of an increase in k is a rise in total output coupled with a fall in scope. Relatively high-cost varieties are discontinued or produced in lower volumes, whereas more is produced of all varieties with average costs or lower.

$\rightarrow$ **“leaner and meaner“-response** of multi-product firms to globalization

Corollary 1: Firm productivity is unaffected by the market-size effect, but rises with the competition effect of an increase in k.

Eckel and Neary (2010)

Globalization and Product Variety

number of varieties per firm $\delta \downarrow$ + number of firms $m \uparrow \rightarrow$ total variety effect?

$N = km\delta$: total number of varieties produced in a symmetric equilibrium

- market-size effect: unaffected
- competition effect: conflicting effects ($m \uparrow$ and $\delta \downarrow$)

$$\frac{d \ln N}{d \ln k} = 1 + \frac{d \ln \delta}{d \ln k} = 1 - \frac{e \delta k m}{\triangle_1} \frac{\alpha(\delta)}{\delta \alpha_\delta}$$

Proposition 5: In partial equilibrium, an increase in the number of countries cannot lower the total number of varieties if the function relating costs to varieties has constant curvature, but it may do so if the technology is sufficiently flexible.

Nocke and Yeaple (2014)

Nocke, Volker and Stephen Yeaple (2014). Globalization and Multiproduct Firms, *International Economic Review* 55(4), pp. 993-1018.

Closed Economy

discrete-time, infinite horizon model with a single (differentiated goods) sector and a single factor of production (labor)

mass L of identical consumers (workers) with a per-period CES utility function:

$$U_s = \left[\int_{\Omega} x_s(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right]^{\frac{\sigma}{\sigma-1}} \tag{12}$$

with $x_s(\omega)$: consumption of product $\omega \in \Omega$ in period s and $\sigma > 1$: elasticity of substitution

in each period: single unit labor supply of each worker ($w_s \equiv 1$)

aggregate demand:

$$X_s(\omega) = A_s p_s(\omega)^{-\sigma} \tag{13}$$

with $p_s(\omega)$: price of product ω in period s and

$$A_s \equiv \frac{L}{\int_\Omega p_s(\omega)^{-(\sigma-1)} d\omega}$$

market entry: irrecoverable setup cost F^e (fraction $F/F^e \in (0,1]$ used to build firm-specific capital equipment, remaining fraction $1 - F/F^e$ spent on intangibles)

upon entry: random draw of its (time-invariant) type $(\widetilde{\theta}, K)$ from $\widetilde{G}$ with support $(0, 1/(\sigma-1)) \times [1, \infty)$

- $\widetilde{\theta}$: organizational efficiency
- K: organizational capital

size of product portfolio (scope): N products (for each product: irrecoverable one-time development cost f)

Nocke and Yeaple (2014)

marginal cost of product ω:

$$c(\omega; k_\omega; \widetilde{\theta}) = \begin{cases} z k_\omega^{-\widetilde{\theta}} & \text{if } \; k_\omega \geq 1 \\ \infty & \text{otherwise} \end{cases}$$

with $z > 0$: cost parameter and k_ω: amount of organizational capital allocated to product ω ($\sum_{\omega \in \mathcal{I}} k_\omega \leq K$ with $\mathcal{I}$: set of products managed by the firm)

at the end of each period: probability of dying: $1 - \beta \in (0, 1)$

sequence in each period:

1. market entry decision
2. scope decision and organizational capital allocation decision
3. pricing decision

Nocke and Yeaple (2014)

monotonic transformation: $\theta \equiv \widetilde{\theta}(\sigma - 1)$ (organizational efficiency)

$\hookrightarrow$ firm type (θ, K) with distribution function G on support $\Theta \equiv (0,1) \times [1, \infty)$

pricing decision:

$$p(\omega; k_\omega; \theta) = \left(\frac{\sigma}{\sigma - 1}\right) c(\omega; k_\omega; \theta)$$

scope decision:

Lemma 1: A firm of type (θ, K) chooses to manage no more than K products, i.e., $N(\theta, K) \leq K$. Moreover, it allocates $k_\omega = K/N(\theta, K)$ units of organizational capital to each one of its $N(\theta, K)$ products. proof

marginal cost of a firm of type (θ, K):

$$c(\theta, K) = z\left(\frac{K}{N(\theta, K)}\right)^{-\frac{\theta}{\sigma - 1}} \quad (14)$$

Nocke and Yeaple (2014)

profit-maximizing price:

$$p(\theta, K) = \left(\frac{\sigma}{\sigma - 1}\right) z \left(\frac{K}{N(\theta, K)}\right)^{-\frac{\theta}{\sigma-1}} \tag{15}$$

per-period profits:

$$\begin{aligned} \pi(\theta, K) &= N(\theta, K)[p(\theta, K) - c(\theta, K)]Ap(\theta, K)^{-\sigma} \\ &= N(\theta, K)(1 - \beta) f \zeta \left(\frac{K}{N(\theta, K)}\right)^{\theta} \end{aligned} \tag{16}$$

with $\zeta \equiv \frac{A}{\sigma(1-\beta)f} \left(\frac{\sigma-1}{\sigma z}\right)^{\sigma-1}$, $A = \frac{(1-\beta)L}{M\left[\int_\Theta N(\theta,K)p(\theta,K)^{-(\sigma-1)} dG(\theta,K)\right]}$ and M: mass of entrants in each period

firm's market value:

$$v(\theta, K) = \frac{\pi(\theta, K)}{1 - \beta} = N(\theta, K) f \zeta \left(\frac{K}{N(\theta, K)}\right)^{\theta} \tag{17}$$

Nocke and Yeaple (2014)

scope decision:

$$\max_{N \geq 0} \; Nf\zeta\left(\frac{K}{N}\right)^{\theta} - Nf \qquad (18)$$

Proposition 1: In equilibrium, a firm of type (θ, K) chooses to manage

$$N(\theta, K) = \begin{cases} K & \text{if } \theta \in (0, \underline{\theta}] \\ K[(1-\theta)\zeta]^{\frac{1}{\theta}} & \text{if } \theta \in [\underline{\theta}, 1) \end{cases} \qquad (19)$$

products, where $\underline{\theta} \equiv (\zeta - 1)/\zeta \in (0, 1)$. proof

market entry decision:

$$\int_{\Theta} v^{e}(\theta, K)dG(\theta, K) - F^{e} = 0 \qquad (20)$$

with $v^{e}(\theta, K) \equiv \frac{\pi(\theta, K)}{1-\beta} - N(\theta, K)f$

labor market equilibrium:

$$L = \frac{M}{1-\beta}\int_{\Theta} N(\theta,K)\left[Ap(\theta,K)^{-\sigma}c(\theta,K)\right] dG(\theta,K) + M\left[f\int_{\Theta} N(\theta,K)dG(\theta,K) + F^{e}\right]$$

$$L = \sigma M\left[f\int_{\Theta} N(\theta,K)dG(\theta,K) + F^{e}\right] \tag{21}$$

equilibrium given by $N(\cdot,\cdot), p(\cdot,\cdot), M, \zeta$ satisfying (15), (16), (18), (19) und (20)

Open economy

decision adjustment: for each product: sales location decision (domestically or domestically and abroad)

with exports: one-time irrecoverable cost f^{x} and iceberg-type trade cost $\tau > 1$

pricing decision:

- domestic price: $p(\omega; k_\omega; \theta) = (\sigma/(\sigma-1))c(\omega; k_\omega; \theta)$
- price abroad: $p^*(\omega; k_\omega; \theta) = \tau p(\omega; k_\omega; \theta)$

Lemma 2: A firm of type (θ, K) chooses to manage no more than K products, i.e., $N(\theta, K) \leq K$. Generically, it exports all of its products, denoted $\delta^x(\theta, K) = 1$, or none, $\delta^x(\theta, K) = 0$. In either case, the firm allocates the same amount $k_\omega = K/N(\theta, K)$ of its organizational capital to each one of its $N(\theta, K)$ products. proof

marginal cost:

$$c(\theta, K) = z\left(\frac{K}{N(\theta, K)}\right)^{-\frac{\theta}{\sigma-1}} \quad (22)$$

price for the domestic and foreign market:

$$p(\theta, K) = \left(\frac{\sigma}{\sigma-1}\right)c(\theta, K) \text{ and } p^*(\theta, K) = \tau\left(\frac{\sigma}{\sigma-1}\right)c(\theta, K) \quad (23)$$

scope, capital allocation and export status decision:

$$\max_{N\in[0,K],\delta^x\in\{0,1\}} N\left[f\zeta(1+\delta^x\rho)\left(\frac{K}{N}\right)^{\theta}-(f+\delta^x f^x)\right] \tag{24}$$

with $\rho\equiv\tau^{-(\sigma-1)}$: measure of trade freeness

assumption:

$$\frac{\ln(1+f^x/f)}{\ln(1+\rho)}>\zeta>1 \tag{25}$$

Proposition 2: In the equilibrium of the open economy, the export decision of a firm of type (θ,K) is given by

$$\delta^x(\theta,K)=\begin{cases}0 & \text{if } \theta\in(0,\theta^x)\\ 1 & \text{if } \theta\in(\theta^x,1)\end{cases} \tag{26}$$

with $\theta^x\equiv 1-\frac{\ln(1+\rho)}{\ln(1+f^x/f)}\in(\underline{\theta},1)$.

Proposition 2 (cont.): The firm's equilibrium number of products is

$$N(\theta, K) = \begin{cases} K & \text{if } \theta \in (0, \underline{\theta}] \\ K((1-\theta)\zeta)^{\frac{1}{\theta}} & \text{if } \theta \in [\underline{\theta}, \theta^x) \\ K\left[\left(\frac{1+\rho}{1+f^x/f}\right)(1-\theta)\zeta\right]^{\frac{1}{\theta}} & \text{if } \theta \in (\theta^x, 1). \end{cases} \quad (27)$$

proof

per-period profit:

$$\pi(\theta, K) = N(\theta, K)(1-\beta) f \zeta [1 + \delta^x(\theta, K)\rho] \left(\frac{K}{N(\theta, K)}\right)^{\theta} \quad (28)$$

market entry decision:

$$\int_{\Theta} v^e(\theta, K) dG(\theta, K) - F^e = 0 \quad (29)$$

with $v^e(\theta, K) = \frac{\pi(\theta, K)}{1-\beta} - N(\theta, K)(f + \delta^x(\theta, K) f^x)$

labor market equilibrium:

$$L = \frac{AM}{1-\beta}\left(\frac{\sigma}{\sigma-1}\right)^{-\sigma}\int_{\Theta}(1+\delta^{x}(\theta,K)\rho)N(\theta,K)c(\theta,K)^{-(\sigma-1)}dG(\theta,K)$$
$$+M\left[\int_{\Theta}(f+\delta^{x}(\theta,K)f^{x})N(\theta,K)dG(\theta,K)+F^{e}\right]$$
$$=\sigma M\left[\int_{\Theta}(f+\delta^{x}(\theta;K)f^{x})N(\theta,K)dG(\theta,K)+F^{e}\right] \quad (30)$$

equilibrium given by $N(\cdot,\cdot), p(\cdot,\cdot), p^{*}(\cdot,\cdot), \delta^{x}(\cdot,\cdot), M, \zeta$ satisfying (21), (22), (25), (26), (27), (28), (29)

Effects of globalization

reduction in the iceberg-type trade cost τ (increase in the trade freeness parameter ρ)

Nocke and Yeaple (2014)

Lemma 3: Consider an increase in trade freeness from ρ to $\rho^{'} > \rho$. This lowers the effective market size facing non-exporters, i.e., $\zeta^{'} < \zeta$, and raises the effective market size facing exporters, i.e., $\zeta^{'}(1+\rho^{'}) > \zeta(1+\rho)$. ▸ proof

Proposition 3: Consider an increase in trade freeness from ρ to $\rho^{'} > \rho$. This induces the thresholds for exporting and for maximal diversification to fall: $\theta^{x'} < \theta^{x}$ and $\underline{\theta}^{'} < \underline{\theta}$.

Proposition 4: Consider an increase in trade freeness from ρ to $\rho^{'} > \rho$. This causes firms that initially sold only domestically to drop products, i.e., $N(\theta, K)^{'} \leq N(\theta, K)$ for all $\theta \in (0, \theta^{x})$, with a strict inequality if $\theta \in (\underline{\theta}^{'}, \theta^{x})$, and all continuing exporters to increase the number of products they manage, i.e., $N(\theta, K)^{'} > N(\theta, K)$ for all $\theta \in (\theta^{x}, 1)$. ▸ proof

Nocke and Yeaple (2014)

Corollary 1: Consider an increase in trade freeness from ρ to $\rho^{'} > \rho$. For firms that initially sold only domestically, this results in higher TFP, i.e., $c(\theta, K)^{'} \leq c(\theta, K)$ for all $\theta \in (0, \theta^{x})$, with a strict inequality if $\theta \in (\underline{\theta}^{'}, \theta^{x})$. For continuing exporters, this results in lower TFP, i.e., $c(\theta, K)^{'} > c(\theta, K)$ for all $\theta \in (\theta^{x}, 1)$.

Appendix

often used terms:

$$\alpha(\delta) = \delta \left[\gamma(\delta) - \mu'_\gamma\right]$$

$$\beta(\delta) = \delta \left[\gamma(\delta)\mu'_\gamma - \mu''_\gamma\right] = \alpha(\delta)\mu'_\delta - \delta\sigma^2_\gamma$$

$$\alpha_\delta = \delta\gamma_\delta$$

$$\beta_\delta = \mu'_\gamma\alpha_\delta$$

Appendix

$$\frac{\partial \pi_j}{\partial x_j(i)} = p_j(i) - c_j(i) + \int_0^{\delta_j} \frac{\partial p_j(i^*)}{\partial x_j(i)} x_j(i^*) di^* = 0$$

$$i = i^* : \frac{\partial p_j(i^*)}{\partial x_j(i)} = -b' \quad \text{and} \quad i \neq i^* : \frac{\partial p_j(i^*)}{\partial x_j(i)} = -b'e$$

$$\frac{\partial \pi_j}{\partial x_j(i)} = p_j(i) - c_j(i) - b' \left[(1-e) x_j(i) + eX_j \right] = 0, \quad X_j \equiv \int_0^{\delta_j} x_j(i^*) di^* \tag{4}$$

back

Appendix

$$\frac{d \ln LP}{d \ln \theta} = \frac{\partial \ln LP}{\partial \ln \theta} + \frac{\partial \ln LP}{\partial \ln \delta}\frac{d \ln \delta}{d \ln \theta}$$

$$l = \psi(\theta)\beta(\delta) \quad \text{and} \quad \frac{\partial \ln l}{\partial \ln \theta} = \frac{\partial \ln l}{\partial \theta}\frac{\partial \theta}{\partial \ln \theta} = \frac{\psi'}{\psi}\theta$$

$$x = \psi(\theta)\left[\gamma(\delta) - \gamma(i)\right] \quad \text{and} \quad \frac{\partial x}{\partial \ln \theta} = \frac{\partial x}{\partial \theta}\frac{\partial \theta}{\partial \ln \theta} = \left[\gamma(\delta) - \gamma(i)\right]\psi'\theta$$

$$\frac{\int_0^\delta h(i)\frac{\partial x}{\partial \ln \theta}di}{\int_0^\delta h(i)x(i)di} = \frac{\int_0^\delta h(i)\psi'(\theta)\theta\left[\gamma(\delta) - \gamma(i)\right]di}{\int_0^\delta h(i)\psi(\theta)\left[\gamma(\delta) - \gamma(i)\right]di} = \frac{\psi'}{\psi}\theta$$

$$\frac{d \ln LP}{d \ln \theta} = \frac{\partial \ln LP}{\partial \ln \delta}\frac{d \ln \delta}{d \ln \theta} \quad \left(\frac{\partial \ln LP}{\partial \ln \theta} = 0\right)$$

back

Appendix

$$\left.\frac{\partial \ln \mathrm{LP}}{\partial \ln \delta}\right|_{h(i)=\gamma(i)} = \frac{\int_0^\delta \gamma(i)\frac{\partial x(i)}{\partial \ln \delta}di}{\int_0^\delta \gamma(i)x(i)di} - \frac{\partial \ln I}{\partial \ln \delta}$$

$$= \frac{1}{I}\int_0^\delta \gamma(i)\frac{\partial x(i)}{\partial \ln \delta}di - \frac{\partial \ln I}{\partial \ln \delta}$$

$$= \frac{1}{I}\int_0^\delta \frac{\partial \gamma(i)x(i)}{\partial \ln \delta}di - \frac{\partial \ln I}{\partial \ln \delta}$$

$$= \int_0^\delta \frac{\partial \ln I}{\partial I}\frac{\partial \gamma(i)x(i)}{\partial \ln \delta}di - \frac{\partial \ln I}{\partial \ln \delta}$$

$$= \frac{\partial \ln I}{\partial \ln \delta}\int_0^\delta \frac{\partial \gamma(i)x(i)}{\partial I}di - \frac{\partial \ln I}{\partial \ln \delta} = 0$$

back

Appendix

$$
\begin{aligned}
\left.\frac{\partial \ln \mathrm{LP}}{\partial \ln \delta}\right|_{h(i)=1} = \frac{\partial \ln X}{\partial \ln \delta} - \frac{\partial \ln I}{\partial \ln \delta} &= \frac{\partial \ln \alpha(\delta)}{\partial \ln \delta} - \frac{\partial \ln \beta(\delta)}{\partial \ln \delta} \\
(\text{since } X = \int_0^\delta \frac{w\left[\gamma(\delta) - \gamma(i)\right]}{2b'(1-e)} di &= \frac{w}{2b'(1-e)} \delta \left(\gamma(\delta) - \mu'_\gamma\right) \\
&= \frac{w\alpha(\delta)}{2b'(1-e)}) \\
&= \frac{\delta \alpha_\delta}{\alpha(\delta)} - \frac{\delta \beta_\delta}{\beta(\delta)} \\
&= -\frac{\delta^2 \alpha_\delta \sigma_\gamma^2}{\alpha(\delta)\beta(\delta)} < 0
\end{aligned}
$$

back

Appendix

$$\left.\frac{\partial \ln \mathrm{LP}}{\partial \ln \delta}\right|_{h(i)=p(i)} = \frac{\int_0^\delta p(i)\frac{\partial x(i)}{\partial \ln \delta}di}{\int_0^\delta p(i)x(i)di} - \frac{\partial \ln I}{\partial \ln \delta}$$

$$\begin{aligned} p(i)x(i) &= \frac{1}{2}\left(a' + w\gamma(i) - b'e(Y-X)\right)\frac{w}{2b'(1-e)}\left[\gamma(\delta)-\gamma(i)\right] \\ &= \frac{1}{2}\left(w\gamma(i) + w\gamma(\delta) + 2b'eX\right)\frac{w}{2b'(1-e)}\left[\gamma(\delta)-\gamma(i)\right] \\ &= w\left(\frac{1}{2}\left(\gamma(i)+\gamma(\delta)\right) + e\frac{\alpha(\delta)}{2(1-e)}\right)\frac{w}{2b'(1-e)}\left[\gamma(\delta)-\gamma(i)\right] \end{aligned}$$

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Appendix

$$
\begin{aligned}
X &= \int_0^{\delta} \frac{a' - w\gamma(i) - b'e(X+Y)}{2b'(1-e)} di \\
&= \frac{1}{2b'(1-e)} \int_0^{\delta} (a' - w\gamma(i) - b'e(1+km)X) di
\end{aligned}
$$

$$
\begin{aligned}
X\left(1 + \frac{b'e\delta(1+km)}{2b'(1-e)}\right) &= \frac{1}{2b'(1-e)} \int_0^{\delta} (a' - w\gamma(i)) di \\
X\left(\frac{2b'(1-e) + b'e\delta(1+km)}{2b'(1-e)}\right) &= \frac{\delta}{2b'(1-e)} \left(a' - w\mu'_{\gamma}\right)
\end{aligned}
$$

$$
X = \frac{\delta}{b'(2(1-e) + e\delta(1+km))} \left(a' - w\mu'_{\gamma}\right)
$$

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Appendix

$$\ln X = \ln\left(\left(a' - w\mu'_\gamma\right)\delta\right) - \ln\left(\left(2(1-e) + e\delta(1+km)\right)b'\right)$$

$$\mu'_\gamma = \frac{1}{\delta}\int_0^\delta \gamma(i)di$$

$$\frac{d}{d\delta}\mu'_\gamma = \frac{\gamma(\delta)\delta - \int_0^\delta \gamma(i)di}{\delta^2} = \frac{\gamma(\delta) - \mu'_\gamma}{\delta}$$

$$\frac{d\ln X}{d\ln\delta} = \frac{\delta}{\left(a' - w\mu'_\gamma\right)\delta}\left(\left(a' - w\mu'_\gamma\right) + \delta\left(-w\frac{\gamma(\delta) - \mu'_\gamma}{\delta}\right)\right) - \frac{\delta}{(2(1-e) + e\delta(1+km))b'}\left(eb'(1+km)\right)$$

$$\frac{d\ln X}{d\ln\delta} = \frac{a' - w\gamma(\delta)}{a' - w\mu'_\gamma} - \frac{eb'(1+km)X}{a' - w\mu'_\gamma} = \frac{a' - w\gamma(\delta) - eb'(1+km)X}{a' - w\mu'_\gamma}$$

▸ back

Appendix - Industry Equilibrium Comparative Statics

$$\begin{bmatrix} \triangle_1 & 0 \\ e(1+km) & \frac{2(1-e)\delta\gamma_\delta}{\alpha(\delta)} \end{bmatrix} \begin{bmatrix} d\ln X \\ d\ln\delta \end{bmatrix} = \begin{bmatrix} \triangle_1 \\ e(1+km) \end{bmatrix} d\ln L$$

$$-ekm \begin{bmatrix} \delta \\ 1 \end{bmatrix} d\ln m + \begin{bmatrix} \triangle_0 \\ e \end{bmatrix} d\ln k - \begin{bmatrix} \delta\mu'_\gamma \\ \gamma(\delta) \end{bmatrix} \frac{2(1-e)}{\alpha(\delta)} d\ln w$$

$$d\ln X = d\ln L - \frac{e\delta km}{\triangle_1} d\ln m + \frac{\triangle_0}{\triangle_1} d\ln k - \frac{2(1-e)\delta\mu'_\gamma}{\triangle_1\ \alpha(\delta)} d\ln w$$

$$d\ln\delta = -\frac{e\delta km\alpha(\delta)}{\triangle_1\ \delta\alpha_\delta}(d\ln m + d\ln k) - \frac{2(1-e)\delta\mu'_\delta + \triangle_1\ \alpha(\delta)}{\triangle_1\ \delta\alpha_\delta} d\ln w$$

back

Appendix - Industry Equilibrium Comparative Statics

$$d \ln x(i) = d \ln L - \frac{ekm\alpha(\delta)}{\Delta_1 \left[\gamma(\delta) - \gamma(i)\right]} d \ln m + \left[\frac{\Delta_0}{\Delta_1} + \left(1 - \frac{\Delta_0}{\Delta_1}\right) \frac{\mu'_\gamma - \gamma(i)}{\gamma(\delta) - \gamma(i)}\right] d \ln k - \frac{2(1-e)\gamma(i) - e\delta(1+km)\left[\mu'_\gamma - \gamma(i)\right]}{\Delta_1 \left[\gamma(\delta) - \gamma(i)\right]} d \ln w$$

back

Appendix - Industry Equilibrium Comparative Statics

$$
\begin{aligned}
0 &= \frac{\Delta_0}{\Delta_1} + \left(1 - \frac{\Delta_0}{\Delta_1}\right) \frac{\mu'_\gamma - \gamma(i)}{\gamma(\delta) - \gamma(i)} \\
(\gamma(i) - \gamma(\delta)) \frac{\Delta_0}{\Delta_1} &= \left(1 - \frac{\Delta_0}{\Delta_1}\right) (\mu'_\gamma - \gamma(i)) \\
\widetilde{\gamma}^{PE} = \gamma(i) &= \frac{\Delta_0}{\Delta_1} \gamma(\delta) + \left(1 - \frac{\Delta_0}{\Delta_1}\right) \mu'_\gamma
\end{aligned}
$$

back

Appendix

Having incurred development cost for $N = \#\mathcal{I}$ products:

$$\max_{\{k_\omega\}_{\omega\in\mathcal{I}}} \frac{1}{1-\beta} \sum_{\omega\in\mathcal{I}^+\equiv\{\omega\in\mathcal{I}|k_\omega\geq 1\}} \left(p(\omega,k_\omega,\theta) - c(\omega,k_\omega,\theta)\right) x(\omega,k_\omega,\theta)$$

$$= \frac{A}{(1-\beta)\sigma}\left(\frac{\sigma-1}{\sigma z}\right)^{\sigma-1} \sum_{\omega\in\mathcal{I}^+\equiv\{\omega\in\mathcal{I}|k_\omega\geq 1\}} (k_\omega)^\theta$$

subject to $\sum_{\omega\in\mathcal{I}} k_\omega \leq K$

objective function increasing and concave in the k_ω's $\rightarrow$ full capital endowment exhaustion and choice of either $k_\omega = k \geq 1$ or $k_\omega = 0$ (latter not a solution!) $\rightarrow$ $N(\theta, K) \leq K$ and $k_\omega = K/N(\theta, K)$

back

Appendix

$$\max_{N \geq 0} \quad Nf\zeta \left(\frac{K}{N}\right)^{\theta} - Nf$$

$$\text{FOC:} \quad f\zeta \left(\frac{K}{N}\right)^{\theta} + Nf\zeta\theta \left(\frac{K}{N}\right)^{\theta-1} \left(-\frac{K}{N^2}\right) - f \stackrel{!}{=} 0$$

$$f\zeta \left(\frac{K}{N}\right)^{\theta} (1-\theta) = f$$

$$(1-\theta) f\zeta K^{\theta} \widetilde{N}(\theta, K)^{-\theta} = f$$

$$\widetilde{N}(\theta, K) = K \left[(1-\theta)\zeta\right]^{\frac{1}{\theta}}$$

$\widetilde{N}(\theta, K) > 0$ for all $(\theta, K) \in \Theta$ and $\widetilde{N}(\theta, K) \leq K$

$$N(\theta, K) = \min\{K, \widetilde{N}(\theta, K)\}$$

back

Appendix (cont.)

$\widetilde{N}(\theta, K)$ strictly decreasing in θ:

$$\frac{\partial \widetilde{N}(\theta, K)}{\partial \theta} \frac{1}{K} = -\frac{\zeta}{\theta^2} \left[(1-\theta)\zeta\right]^{\frac{1-\theta}{\theta}} \underbrace{\left(\theta + (1-\theta)\ln((1-\theta)\zeta)\right)}_{\equiv \Psi(\theta)}$$

$$\frac{\partial \widetilde{N}(\theta, K)}{\partial \theta} < 0 \quad \Leftrightarrow \quad \Psi(\theta) > 0$$

$\Psi(0) = \ln(\zeta) > 0$ as $\zeta > 1$ by assumption

$$\Psi'(\theta) = -\ln((1-\theta)\zeta) \text{ and } \Psi''(\theta) = \frac{1}{1-\theta} > 0$$

unique minimum: $\Psi'(\theta^m) = 0 \Leftrightarrow \theta^m = 1 - \frac{1}{\zeta} = \frac{\zeta - 1}{\zeta}$ with $\Psi(\theta^m) = \theta^m > 0$

$$\widetilde{N}(\underline{\theta}, K) = K \quad \Leftrightarrow \quad \underline{\theta} = \frac{\zeta - 1}{\zeta}$$

back

Appendix

Having incurred development cost and given exporting decision:

$$\max_{\{k_\omega\}_{\omega\in\mathcal{I}}} \zeta f \sum_{\omega\in\mathcal{I}^+\equiv\{\omega\in\mathcal{I}|k_\omega\geq 1\}} [1+\chi(\omega)\rho](k_\omega)^\theta$$

subject to $\sum_{\omega\in\mathcal{I}} k_\omega \leq K$ ($\chi(\omega) = 1$ if ω is exported, $\chi(\omega) = 0$ otherwise)

objective function increasing and concave in k_ω's $\rightarrow$ full capital endowment exhaustion and choice of $k_\omega \in \{0, k^x\}$ with $k^x \geq 1$ if $\chi(\omega) = 1$ and $k_\omega \in \{0, k^d\}$ with $k^d \geq 1$ if $\chi(\omega) = 0$ (allocation of zero organizational capital not optimal!) $\rightarrow N(\theta, K) \leq K$

Lagrangian:

$$\mathcal{L} = f\zeta N[(1-\delta)(k^d)^\theta + \delta(1+\rho)(k^x)^\theta] - \lambda N\left[(1-\delta)k^d + \delta k^x - \frac{K}{N}\right]$$

with δ as the share of exported products and λ as the Lagrange multiplier on the firm's organizational capital constraint $\rightarrow$ Lagrangian linear in $\delta \rightarrow$ $\delta \in \{0, 1\}$ back

Appendix

$\delta^x = 0$: (24) simplifies to (18): $N^d(\theta, K) = \min\{K, K[(1-\theta)\zeta]^{\frac{1}{\theta}}\}$

$\delta^x = 1$:

$$\max_{N \in [0,K]} \quad N\left[f\zeta(1+\rho)\left(\frac{K}{N}\right)^{\theta} - (f + f^x)\right]$$

$$\text{FOC:} \quad \left[f\zeta(1+\rho)\left(\frac{K}{N}\right)^{\theta} - (f+f^x)\right] - \theta f\zeta(1+\rho)\left(\frac{K}{N}\right)^{\theta} \overset{!}{=} 0$$

$$N^x(\theta, K) = \min\{K, K\left[\left(\frac{1+\rho}{1+f^x/f}\right)(1-\theta)\zeta\right]^{\frac{1}{\theta}}\}$$

optimal choice: $\delta^x(\theta, K) = 0$ if $v^d(\theta, K) > v^x(\theta, K)$ and $\delta^x(\theta, K) = 1$ if $v^d(\theta, K) < v^x(\theta, K)$

$$v^d(\theta, K) = \max\{Kf[\zeta - 1], Kf[(1-\theta)\zeta]^{1/\theta}(\theta/(1-\theta))\}$$

back

Appendix (cont.)

$$v^x(\theta, K) = Kf\left[\left(\frac{1+\rho}{1+f^x/f}\right)(1-\theta)\zeta\right]^{\frac{1}{\theta}}\left(1+\frac{f^x}{f}\right)\left(\frac{\theta}{1-\theta}\right)$$

$$v^x(\theta, K) > Kf[(1-\theta)\zeta]^{1/\theta}(\theta/(1-\theta)) \Leftrightarrow \left(\frac{1+\rho}{1+f^x/f}\right)^{\frac{1}{\theta}}(1+f^x/f) > 1$$

or

$$\theta > \theta^x \equiv 1 - \frac{\ln(1+\rho)}{\ln(1+f^x/f)} > \underline{\theta}$$

$$Kf[(1-\theta)\zeta]^{1/\theta}(\theta/(1-\theta)) > Kf[\zeta-1] \Leftrightarrow \theta > \underline{\theta}$$

$$v^x(\theta, K) > v^d(\theta, K) \Leftrightarrow \theta > \theta^x$$

back

Appendix

Inserting $\pi(\theta, K)$ and $N(\theta, K)$ into (29):

$$f \int_1^{\infty} K \left\{ (\zeta - 1) \int_0^{\underline{\theta}} g(\theta, K) d\theta + \int_{\underline{\theta}}^{\theta^x} \left(\frac{\theta}{1-\theta} \right) [(1-\theta)\zeta]^{\frac{1}{\theta}} g(\theta, K) d\theta \right.$$

$$\left. + \int_{\theta^x}^{1} \left(\frac{\theta(1 + f^x/f)}{1-\theta} \right) \left[\left(\frac{1+\rho}{1+f^x/f} \right) (1-\theta)\zeta \right]^{\frac{1}{\theta}} g(\theta, K) d\theta \right\} dK - F^e = 0$$

$$\frac{d\zeta}{\zeta} f \int_1^{\infty} K \left\{ \int_0^{\underline{\theta}} \zeta g(\theta, K) d\theta + \int_{\underline{\theta}}^{\theta^x} \left(\frac{1}{1-\theta} \right) [(1-\theta)\zeta]^{\frac{1}{\theta}} g(\theta, K) d\theta \right.$$

$$\left. + \int_{\theta^x}^{1} \left(\frac{1 + f^x/f}{1-\theta} \right) \left[\left(\frac{1+\rho}{1+f^x/f} \right) (1-\theta)\zeta \right]^{\frac{1}{\theta}} g(\theta, K) d\theta \right\} dK$$

$$+ \frac{d\rho}{(1+\rho)} f \int_1^{\infty} K \int_{\theta^x}^{1} \left(\frac{1 + f^x/f}{1-\theta} \right) \left[\left(\frac{1+\rho}{1+f^x/f} \right) (1-\theta)\zeta \right]^{\frac{1}{\theta}} g(\theta, K) d\theta dK$$

$$= 0$$

back

Appendix

$d\zeta d\rho < 0$ and $\zeta^{'} < \zeta$

suppose: $\zeta(1+\rho)$ decreasing as well $\rightarrow \zeta^{'}(1+\rho^{'}) \leq \zeta(1+\rho)$, then LHS of (29) negative after trade liberalization (contradiction!) $\rightarrow \zeta^{'}(1+\rho^{'}) > \zeta(1+\rho)$

back

Appendix

firm (θ, K) with $\theta \in (0, \underline{\theta}']$: $N(\theta, K)' = N(\theta, K) = K$ if $\theta \in (0, \underline{\theta}']$ and $N(\theta, K)' < N(\theta, K) = K$ if $\theta \in (\underline{\theta}', \underline{\theta}]$

firm (θ, K) with $\theta \in (\underline{\theta}, \theta^{x'}) \cup (\theta^x, 1)$: $N(\theta, K)' < N(\theta, K)$ if $\theta \in (\underline{\theta}, \theta^{x'})$ and $N(\theta, K)' > N(\theta, K)$ if $\theta \in (\theta^x, 1)$

firm (θ, K) with $\theta \in (\theta^{x'}, \theta^x)$:

$$\frac{N(\theta, K)'}{N(\theta, K)} = \left(\frac{\zeta'}{\zeta}\right)^{\frac{1}{\theta}} \left(\frac{1 + \rho'}{1 + f^X/f}\right)^{\frac{1}{\theta}} < 1$$

back