

- The standard DFS does not generalize to more than two countries, because there is no clean way to define the “chain of comparative advantage”.
- **Eaton and Kortum (2002)** generalizes DFS to a situation with many countries that incorporates a role for geography.
- It does so using a probabilistic approach to technology.
- Goal is to (i) propose a theoretical model, (ii) structurally estimate the parameters using cross-country data on trade and prices, and (iii) run counterfactual exercises to understand the effect of trade liberalization, technological progress, etc.

- Continuum of goods j located on unit interval $[0,1]$.
- N countries indexed by i .
- Input cost (labor + intermediates) in country i denoted by c_i .
- Goods are homogeneous and there is perfect competition.
- Efficiency of country i in producing good j is $z_i(j)$.
- Cost of producing a unit of good j in country i is $c_i/z_i(j)$.

- Geographic barriers are of the iceberg form. To deliver 1 unit from country i to country j requires shipping $d_{ni} > 1$ units.
- The price of a good j produced in country i and bought in country j is

$$p_{ni}(j) = \left(\frac{c_i}{z_i(j)}\right) d_{ni} \quad (1)$$

- Actual price paid by consumers in country n for good j (after shopping around) is

$$p_n(j) = \min\{p_{ni}(j); i = 1, \dots, N\} \quad (2)$$

- Preferences are Cobb-Douglas

$$U = \left[\int_0^1 Q(j)^{(\sigma-1)/\sigma} dj\right]^{\sigma/(\sigma-1)} \quad (3)$$

- The efficiency of country i in the production of good j is the realization of a random variable, Z_i , drawn from a Fréchet distribution

$$F_i(z) = Pr[Z_i \leq z] = e^{-T_i z^{-\theta}} \quad (4)$$

where $T_i > 0$ and $\theta > 1$.

- Two parameters: T_i is country-specific and determines absolute advantage (higher T_i implies greater absolute advantage); and θ , common to all countries, determines the variation within the distribution, and thus the scope of comparative advantage.

- Since the productivity to produce good j in country i is the realization of a random variable, the price at which i offer good j in country n is also the realization of a random variable, $c_i d_{ni} / Z_i$.
- This implies that each country i presents country n with a price distribution, $G_{ni}(p) = Pr[P_{ni} \leq p]$. The probability that $P_{ni} \leq p$ is the same as the probability that $c_i d_{ni} / Z_i$ is less than p , which is the same as the probability that Z_i is more than $c_i d_{ni} / p$. This implies that

$$G_{ni}(p) = Pr[P_{ni} \leq p] = 1 - F_i(c_i d_{ni} / p) \quad (5)$$

- The actual price distribution in country n is then:

$$G_n(p) = Pr[P_n \leq p] = 1 - \prod_i^N (1 - G_{ni}(p)) \quad (6)$$

i.e., 1 minus the probability that the price from all source countries is greater than p .

- By substituting (5) into (6) we can re-write this expression as

$$G_n(p) = 1 - e^{-\Phi_n p^\theta} \quad \text{where } \Phi_n = \sum_{i=1}^N T_i (c_i d_{ni})^{-\theta} \quad (7)$$

It can be shown that trading with more countries or technological improvement lowers prices, whereas greater geographical barriers increases prices.

Properties of price distributions

- *Property (a)* The probability of country i providing a good at the lowest price to country n is

$$\pi_{ni} = (T_i(c_i d_{ni})^{-\theta}) / \Phi_n \quad (8)$$

Because of law of large numbers, π_{ni} is also the fraction of goods i sells in n .

- *Property (b)* The price of a good that country n actually buys from any country i also has the distribution $G_n(p)$. That is, the price distribution of the goods sold by country i to country n is independent of i . Countries with better access or better technology sell a wider range of goods (increase in the extensive margin), such that the distribution of the prices of what it sells in country n is the same as the overall price distribution in country n .

Properties of price distributions

- *Property (c)* The price index for the CES utility function can be shown to be

$$p_n = \gamma \Phi_n^{-1/\theta} \quad \text{where } \gamma = \left[\Gamma\left(\frac{\theta + 1 - \sigma}{\theta}\right) \right]^{1/(1-\sigma)} \quad (9)$$

where Γ is the Gamma function.

- Property (b) implies that country n 's average expenditure per good does not vary by source country, so that the fraction of goods country n buys from country i is also the fraction of its expenditure on goods from i :

$$\pi_{ni} = \frac{X_{ni}}{X_n} = \frac{T_i(c_i d_{ni})^{-\theta}}{\Phi_n} = \frac{T_i(c_i d_{ni})^{-\theta}}{\sum_{k=1}^N T_k(c_k d_{nk})^{-\theta}} \quad (10)$$

- This is similar to a standard gravity equation: value of exports from i to n depends positively on total expenditure of n and negatively on geographic barriers.

Trade flows and gravity

- Country i 's total exports are:

$$Q_i = \sum_{m=1}^N X_{mi} = T_i c_i^{-\theta} \sum_{m=1}^N \frac{d_{mi}^{-\theta} X_m}{\Phi_m} \quad (11)$$

- Combining (10) and (11) gives us

$$X_{ni} = \frac{\left(\frac{d_{ni}}{p_n}\right)^{-\theta} X_n}{\sum_{m=1}^N \left(\frac{d_{mi}}{p_m}\right)^{-\theta} X_m} Q_i \quad (12)$$

- Hence, exports from i to n depend on (i) total expenditure of n , X_n ; (ii) total sales of exporter, Q_i ; and (iii) geographic barriers.
- Numerator $\left(\left(\frac{d_{ni}}{p_n}\right)^{-\theta} X_n\right)$ can be interpreted as the market size of destination n as perceived by exporter i , whereas the denominator $\left(\sum_{m=1}^N \left(\frac{d_{mi}}{p_m}\right)^{-\theta} X_m\right)$ can be interpreted as the total world market as perceived by exporter i .

Closing the model

- To close model, we need to determine c_i , which has been taken as given until now. Income of inputs (labor+intermediates) must equal spending by inputs (labor+intermediates).
- See paper for further details.

Parameter estimates and counterfactuals

- Use data on 19 OECD countries to estimate parameter values.
- Run counterfactuals.
- Example: quantify effects of declining geographic barriers.
- Example: quantify effect of technological progress in one country on welfare in others.