Economies of Scope and Trade

Niklas Herzig

Bielefeld University

June 16, 2015

Overview

1. Literature overview
2. Eckel and Neary (2010)
3. Eckel et al. (2015)

Literature overview

Product symmetry	**Product asymmetry**
Products symmetric on both the demand and supply side	*Products asymmetric on the demand side*
Allanson and Montagna (IJIO 2005)	Bernard, Redding and Schott (AER 2010, QJE 2011)
Nocke and Yeaple (IER 2014)	
	Products asymmetric on the supply (cost) side
	Arkolakis, Ganapati and Muendler (2015)
	Mayer, Melitz and Ottaviano (AER 2014)
Cannibalization	
Ju (RIE 2003)	Eckel and Neary (RES 2010)
Feenstra and Ma (2008)	
Baldwin and Gu (2009)	Eckel, Iacovone, Javorcik and Neary (JIE 2015)
Dhingra (AER 2013)	Qiu and Zhou (JIE 2013)

Eckel, Carsten and Peter Neary (2010). Multi-Product Firms and Flexible Manufacturing in the Global Economy, *Review of Economic Studies* 77(1), pp. 188-217.

Preferences and Demand
two-tier utility function:

$$U[u(z)] = \int_0^1 u(z)dz \tag{1}$$

with

$$u(z) = a\int_0^N q(i)di - \frac{1}{2}b\left[(1-e)\int_0^N q(i)^2 di + e\left\{\int_0^N q(i)di\right\}^2\right]$$

$q(i)$: consumption of (horizontally diff.) product variety i, $i \in [0, N]$ and
N: measure of diff. varieties produced in each industry z, $z \in [0, 1]$

Eckel and Neary (2010)

utility maximization problem:

$$\max_{q(i)} \quad U[u(z)] \quad \text{subject to}$$

$$\int_0^1 \int_0^N p(i)q(i)didz \leq I$$

$p(i)$: price of variety i and I: individual income

FOC: inverse individual demand function:

$$\lambda p(i) = a - b\left[(1-e)q(i) + e\int_0^N q(i)di\right] \tag{2}$$

λ: Lagrange multiplier (consumer's marginal utility of income)

L (homogeneous) consumers in each of k identical countries, integrated goods markets and free trade (single variety price worldwide)

Eckel and Neary (2010)

market demand for variety i: $x(i) = kLq(i)$

inverse world market demand function:

$$p(i) = a' - b'[(1-e)x(i) + eY] \quad (3)$$

$a' \equiv a/\lambda$, $b' \equiv b/\lambda kL$ and $Y \equiv \int_0^N x(i)di$: industry output

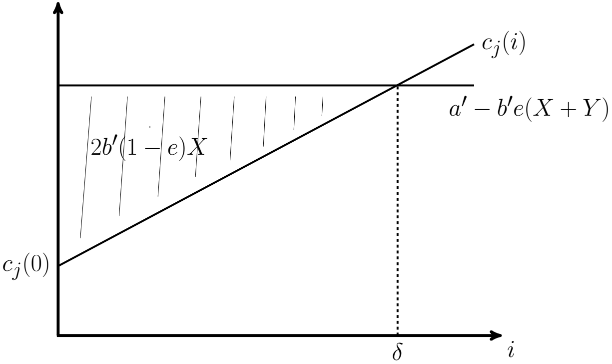
Production and Supply

"flexible manufacturing" technology (core competence):

$c_j(i)$: marginal cost of firm j to produce variety i (independent of output, but different across products: $c_j' > 0$ and $c_j(0) = c_j^0$; e.g. linear: $c_j(i) = c_j^0 + \gamma i$)

Eckel and Neary (2010)

Figure 1



Eckel and Neary (2010)

single-stage Cournot game

profit maximization problem:

$$\max_{x_j(i)} \quad \pi_j = \int_0^{\delta_j} \left[p_j(i) - c_j(i)\right] x_j(i) - F$$

δ_j: mass of products produced (scope) and F: fixed cost

FOC: (i) scale

$$\frac{\partial \pi_j}{\partial x_j(i)} = p_j(i) - c_j(i) - b' \left[(1-e)x_j(i) + eX_j\right] = 0 \qquad (4)$$

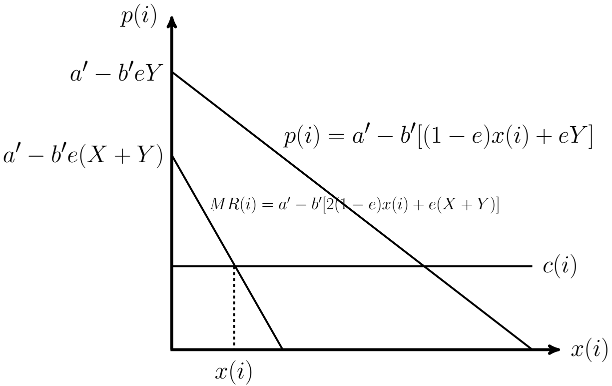
proof

$X_j \equiv \int_0^{\delta_j} x_j(i)di$: firm's aggregate output

$$x_j(i) = \frac{a' - c_j(i) - b'e\left(X_j + Y\right)}{2b'(1-e)} \qquad (5)$$

Eckel and Neary (2010)

Figure 2



Eckel and Neary (2010)

$$p_j(i) = \frac{1}{2}\left[a' + c_j(i) - b'e(Y - X_j)\right] \tag{6}$$

(ii) scope

$$\frac{\partial \pi_j}{\partial \delta_j} = [p_j(\delta_j) - c_j(\delta_j)]\, x_j(\delta_j) = 0 \tag{7}$$

product range: output of the marginal variety (δ_j) zero: $x_j(\delta_j) = 0$

$$c_j(\delta_j) = a' - b'e\,(X_j + Y) \tag{8}$$

$$p_j(\delta_j) = a' - b'eY$$

labour productivity (LP) of multi-product firms:

labour as the only factor of production and economy-wide and perfectly competitive labour market

unit cost of producing each variety:

$$c(i) = w\gamma(i)$$

Eckel and Neary (2010)

total labour input:

$$l = \int_0^\delta \gamma(i)x(i)di$$

$$\frac{d\ln \mathrm{LP}}{d\ln\theta} = \frac{\int_0^\delta h(i)\frac{dx(i)}{d\ln\theta}di}{\int_0^\delta h(i)x(i)di} - \frac{d\ln l}{d\ln\theta} \quad (9)$$

θ: any exogenous variable and $h(i)$: weight of variety i

$$x(i) = \frac{w\left[\gamma(\delta) - \gamma(i)\right]}{2b'(1-e)}$$

$$l = \frac{w\beta(\delta)}{2b'(1-e)} \quad \text{with} \quad \beta(\delta) \equiv \int_0^\delta \gamma(i)\left[\gamma(\delta) - \gamma(i)\right]di$$

$$\frac{d\ln \mathrm{LP}}{d\ln\theta} = \frac{\partial\ln \mathrm{LP}}{\partial\ln\delta}\frac{d\ln\delta}{d\ln\theta}$$

proof

Eckel and Neary (2010)

choice of weights $h(i)$:

$$\left.\frac{\partial \ln \mathrm{LP}}{\partial \ln \delta}\right|_{h(i)=\gamma(i)} = \frac{\int_0^\delta \gamma(i) \frac{\partial x(i)}{\partial \ln \delta} di}{\int_0^\delta \gamma(i) x(i) di} - \frac{\partial \ln l}{\partial \ln \delta} = 0$$ proof

Proposition 1: With given technology, any shock which raises the product range δ
(a) leaves LP unchanged when output changes are marginal cost-weighted,
(b) reduces LP when output is a simple aggregate proof and
(c) reduces LP but by less when output changes are price-weighted proof.

Eckel and Neary (2010)

Industry Equilibrium

symmetric Cournot oligopoly with an exogenously given number of firms m in each of k countries

industry output: $Y = kmX$

FOC for scope (rewrite (8)):

$$w\gamma(\delta) = a' - e(1 + km)b'X \;\Rightarrow\; \text{scope: } \delta(X)$$

FOC for scale (integrate over (5)):

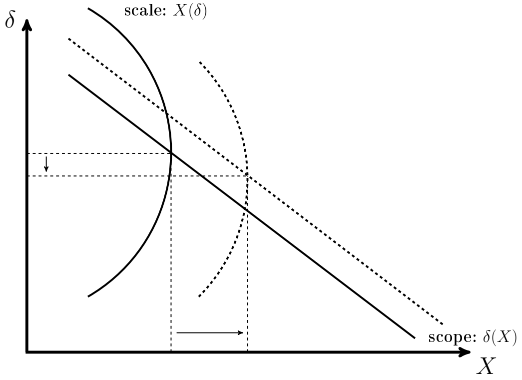
$$X = \frac{\left(a' - w\mu'_\gamma\right)\delta}{\triangle_1\; b'} \text{ with } \triangle_1 \equiv 2(1-e)+e\delta(1+km) > 0 \text{ proof } \Rightarrow \text{ scale: } X(\delta)$$

with $\mu'_\gamma \equiv \frac{1}{\delta}\int_0^\delta \gamma(i)di$

$$\frac{d\ln X}{d\ln\delta} = \frac{a' - w\gamma(\delta) - e(1 + km)b'X}{a' - w\mu'_\gamma}$$

Eckel and Neary (2010)

Figure 3



Eckel and Neary (2010)

Effects of Globalization

globalization: increase in the number of countries k participating in the global economy

two channels:

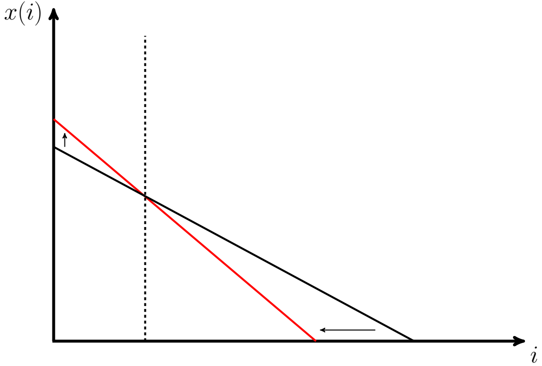
- market-size effect ($L \uparrow$)
- competition effect ($m \uparrow$)

Proposition 2: The *market-size effect* of an increase in k is an equi-proportionate increase in the output of each variety and of total output, but no change in firm scope.

Proposition 3: The *competition effect* of an increase in k is a uniform absolute fall in the output of each variety, coupled with falls in both total firm output and firm scope, but a rise in industry output.

Eckel and Neary (2010)

Figure 4



Eckel and Neary (2010)

full effect:

- on firm output:

$$\frac{d \ln X}{d \ln k} = 1 - \frac{e\delta km}{\triangle_1} \tag{10}$$

where $\triangle_1 - e\delta km = \triangle_0 \; (\equiv 2(1-e) + e\delta) > 0$

- on variety output:

$$\frac{d \ln x(i)}{d \ln k} = 1 - \frac{ekm\alpha(\delta)}{\triangle_1 \left[\gamma(\delta) - \gamma(i)\right]} = \frac{\triangle_0}{\triangle_1} + \left(1 - \frac{\triangle_0}{\triangle_1}\right) \frac{\mu'_\gamma - \gamma(i)}{\gamma(\delta) - \gamma(i)} \tag{11}$$

$\widetilde{\gamma}^{PE}$: labour requirement of the threshold variety whose output is unchanged

$$\widetilde{\gamma}^{PE} = \frac{\triangle_0}{\triangle_1}\gamma(\delta) + \left(1 - \frac{\triangle_0}{\triangle_1}\right)\mu'_\gamma$$

▸ proof

Eckel and Neary (2010)

Proposition 4: The *total effect* of an increase in k is a rise in total output coupled with a fall in scope. Relatively high-cost varieties are discontinued or produced in lower volumes, whereas more is produced of all varieties with average costs or lower.

$\rightarrow$ **"leaner and meaner"-response** of multi-product firms to globalization

Corollary 1: Firm productivity is unaffected by the market-size effect, but rises with the competition effect of an increase in k.

Globalization and Product Variety

number of varieties per firm $\delta \downarrow$ + number of firms $m \uparrow \rightarrow$ total variety effect?

$N = km\delta$: total number of varieties produced in a symmetric equilibrium

- market-size effect: unaffected
- competition effect: conflicting effects ($m \uparrow$ and $\delta \downarrow$)

$$\frac{d \ln N}{d \ln k} = 1 + \frac{d \ln \delta}{d \ln k} = 1 - \frac{e\delta km}{\triangle_1} \frac{\alpha(\delta)}{\delta \alpha_\delta}$$

Proposition 5: In partial equilibrium, an increase in the number of countries cannot lower the total number of varieties if the function relating costs to varieties has constant curvature, but it may do so if the technology is sufficiently flexible.

Eckel et al. (2015)

Eckel, Carsten, Leonardo Iacovone, Beata Javorcik and Peter Neary (2015). Multi-Product Firms at Home and Away: Cost- versus Quality-based Competence, *Journal of International Economics* 95(2), pp. 216-232.

- determinants of economic success of firms (exporters):
  1. firm productivity (among others, Melitz (Econ 2003))
  2. product quality (among others, Manova and Zhang (QJE 2012))
- two views opposed? No, focus on: ïntra-firm extensive margin"
- model of multi-product firms with an endogenous choice of product quality
- extension of the "flexible-manufacturing" model by Eckel and Neary (RES 2010) to investment in quality
- simplification: single monopoly firm (possible: Cournot competition in a heterogeneous-firm industry)

Eckel et al. (2015)

Preferences for quantity and quality

- single market (L consumers)
- representative consumer: quadratic sub-utility function

$$
\begin{aligned}
u &= u_1 + \beta u_2 \\
u_1 &= a^0 Q - \frac{1}{2} b \left[(1-e) \int_{i \in \widetilde{\Omega}} q(i)^2 di + e Q^2 \right] \\
u_2 &= \int_{i \in \widetilde{\Omega}} q(i) \widetilde{z}(i) di
\end{aligned}
\tag{12}
$$

- $\widetilde{\Omega}$: set of differentiated products, $q(i)$: consumption of variety i, $Q \equiv \int_{i \in \widetilde{\Omega}} q(i) di$ and e: substitution index between goods ($0 \leq e \leq 1$)
- $\widetilde{z}(i)$: perceived quality (premium) of variety i

Product Demand

- optimization problem: max u subject to budget constraint $\int_{i\in\widetilde{\Omega}} p(i)q(i)di = I$ (I: individual expenditure on $\widetilde{\Omega}$)
- market inverse demand functions (market-clearing: $x(i) = Lq(i)$):

$$\begin{aligned} p(i) &= a(i) - \widetilde{b}\left[(1-e)x(i) + eX\right] \quad i \in \Omega \subset \widetilde{\Omega} \\ a(i) &= a^0 + \beta\widetilde{z}(i) \\ \widetilde{b} &\equiv \frac{b}{L} \end{aligned} \tag{13}$$

- $X \equiv \int_{i\in\Omega} x(i)di$ (Ω: set of goods actually consumed)

Eckel et al. (2015)

Cost-based Competence

- ignoring quality ($\beta = 0$)
- optimization problem: max π

$$\pi = \int_{i \in \Omega} [p(i) - c(i) - t]\, x(i) di \qquad (14)$$

- t: (uniform) trade cost
- "flexible manufacturing" technology
  1. marginal production costs are independent of output but differ across products: $c(i)$
  2. marginal production cost rise as the firm moves away from its "core competence" variety: $c'(i) > 0 \quad (c(0) = c^0)$

Scale and Scope

1. scale $x(i)$:

$$x(i) = \frac{a(i) - c(i) - t - 2\widetilde{b}eX}{2\widetilde{b}(1-e)} \quad i \in \Omega \tag{15}$$

2. scope δ:

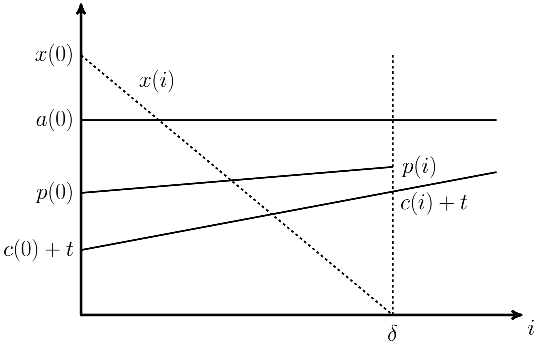
$$x(\delta) = 0$$

3. price $p(i)$:

$$p(i) = \frac{1}{2}\left[a(i) + c(i) + t\right] \tag{16}$$

Eckel et al. (2015)

Figure 5: Profiles of outputs, prices and costs with cost-based competence



Eckel et al. (2015)

Quality-based Competence

- considering quality ($\beta > 0$)
- perceived quality (premium) of variety i:

$$\tilde{z}(i) = (1 - e)z(i) + e\overline{Z} \tag{17}$$

- $z(i)$: variety-specific perceived quality and $\overline{Z}$: perceived quality of the firm's brand ($\overline{Z} \neq \int_{i \in \Omega} z(i)di$)
- (linear-quadratic) specification for the costs of and returns to *investment in quality*:
  1. investment in quality of variety i, $k(i)$: costs: $\gamma k(i)$ and benefits: $z(i) = 2\theta k(i)^{0.5}$
  2. investment in quality of the brand, $\overline{K}$: costs: $\Gamma\overline{K}$ and benefits: $\overline{Z} = 2\Theta\overline{K}^{0.5}$

Eckel et al. (2015)

- optimization problem: max Π

$$\Pi = \int_{i \in \Omega} [(p(i) - c(i) - t)\, x(i) - \gamma k(i)]\, di - \Gamma \overline{K} \tag{18}$$

- FOCs for scale and scope unchanged
- FOCs for investment:

$$(i) \quad \gamma k(i)^{0.5} = \beta(1-e)\theta x(i) \quad i \in [0, \delta]\,; \quad (ii) \quad \Gamma \overline{K}^{0.5} = \beta e \Theta X \tag{19}$$

- comparison: total investment in quality of individual varieties ($K \equiv \int_0^\delta k(i) di$) and investment in brand quality ($\overline{K}$):

$$\frac{K}{\overline{K}} = \left(\frac{1-e}{e} \frac{\theta}{\Theta} \frac{\Gamma}{\gamma} \right)^2 \Phi \quad \text{where} \quad \Phi \equiv \frac{\int_0^\delta x(i)^2 di}{X^2} \tag{20}$$

Eckel et al. (2015)

- scale $x(i)$:

$$x(i) = \frac{a^0 - c(i) - t - 2\left(\widetilde{b} - \overline{\eta}e\right) eX}{2\left[\widetilde{b} - \eta(1-e)\right](1-e)} \quad i \in [0,\delta] \quad \eta \equiv \frac{\beta^2\theta^2}{\gamma} \quad (21)$$

$$\overline{\eta} \equiv \frac{\beta^2\Theta^2}{\Gamma}$$

- η, $\overline{\eta}$: "marginal effectiveness of investment"
- scale $x(i)$:

$$x(i) = \frac{c(\delta) - c(i)}{2\left[\widetilde{b} - \eta(1-e)\right](1-e)} \quad i \in [0,\delta] \quad (22)$$

Eckel et al. (2015)

- price $p(i)$:

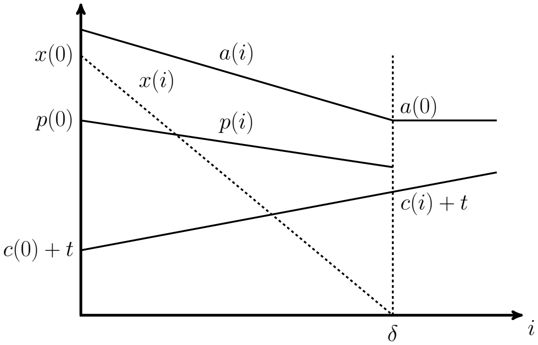
$$p(i) = \frac{\widetilde{b} - 2\eta(1-e)}{2\left[\widetilde{b} - \eta(1-e)\right]} c(i) + \frac{\widetilde{b}}{2\left[\widetilde{b} - \eta(1-e)\right]} c(\delta) + t + \widetilde{b} e X \quad (23)$$

$$i \in [0, \delta]$$

- **Proposition:**
  1. $\widetilde{b} > 2\eta(1-e)$: *cost-based competence* dominates (price rises with i)
  2. $\widetilde{b} < 2\eta(1-e)$: *quality-based competence* dominates (price falls with i)
- quality-based competence more likely to dominate:
  1. when investment in quality is more effective (η larger)
  2. when market size L is larger
  3. when products are more differentiated (e smaller)

Eckel et al. (2015)

Figure 6: Profiles of outputs, prices and costs with quality-based competence



Eckel et al. (2015)

Comparative Statics

Evaluate (10) at $i = \delta$: $$c(\delta) = a^0 - t - 2(\widetilde{b} - \overline{\eta} e)eX \tag{24}$$

Integrate (11) over i: $$X = \frac{\int_0^\delta [c(\delta) - c(i)]\, di}{2\left[\widetilde{b} - \eta(1-e)\right](1-e)} \tag{25}$$

Increase in:	$\overline{\eta}$	η	t	L
X	+	+	-	+
$x(0)$	+	+	-	+
δ	+	-	-	+/-

Appendix

often used terms:

$$\alpha(\delta) = \delta\left[\gamma(\delta) - \mu'_\gamma\right]$$

$$\beta(\delta) = \delta\left[\gamma(\delta)\mu'_\gamma - \mu''_\gamma\right] = \alpha(\delta)\mu'_\delta - \delta\sigma^2_\gamma$$

$$\alpha_\delta = \delta\gamma_\delta$$

$$\beta_\delta = \mu'_\gamma\alpha_\delta$$

Appendix

$$\frac{\partial \pi_j}{\partial x_j(i)} = p_j(i) - c_j(i) + \int_0^{\delta_j} \frac{\partial p_j(i^*)}{\partial x_j(i)} x_j(i^*) di^* = 0$$

$$i = i^* : \frac{\partial p_j(i^*)}{\partial x_j(i)} = -b' \quad \text{and} \quad i \neq i^* : \frac{\partial p_j(i^*)}{\partial x_j(i)} = -b'e$$

$$\frac{\partial \pi_j}{\partial x_j(i)} = p_j(i) - c_j(i) - b' \left[(1-e) x_j(i) + eX_j \right] = 0, \quad X_j \equiv \int_0^{\delta_j} x_j(i^*) di^* \tag{4}$$

back

Appendix

$$\frac{d \ln \mathrm{LP}}{d \ln \theta} = \frac{\partial \ln \mathrm{LP}}{\partial \ln \theta} + \frac{\partial \ln \mathrm{LP}}{\partial \ln \delta} \frac{d \ln \delta}{d \ln \theta}$$

$$I = \psi(\theta)\beta(\delta) \quad \text{and} \quad \frac{\partial \ln I}{\partial \ln \theta} = \frac{\partial \ln I}{\partial \theta} \frac{\partial \theta}{\partial \ln \theta} = \frac{\psi'}{\psi}\theta$$

$$x = \psi(\theta)\left[\gamma(\delta) - \gamma(i)\right] \quad \text{and} \quad \frac{\partial x}{\partial \ln \theta} = \frac{\partial x}{\partial \theta} \frac{\partial \theta}{\partial \ln \theta} = \left[\gamma(\delta) - \gamma(i)\right]\psi'\theta$$

$$\frac{\int_0^\delta h(i) \frac{\partial x}{\partial \ln \theta} di}{\int_0^\delta h(i) x(i) di} = \frac{\int_0^\delta h(i)\psi'(\theta)\theta\left[\gamma(\delta) - \gamma(i)\right] di}{\int_0^\delta h(i)\psi(\theta)\left[\gamma(\delta) - \gamma(i)\right] di} = \frac{\psi'}{\psi}\theta$$

$$\frac{d \ln \mathrm{LP}}{d \ln \theta} = \frac{\partial \ln \mathrm{LP}}{\partial \ln \delta} \frac{d \ln \delta}{d \ln \theta} \qquad \left(\frac{\partial \ln \mathrm{LP}}{\partial \ln \theta} = 0 \right)$$

back

Appendix

$$
\begin{aligned}
\left.\frac{\partial \ln \mathrm{LP}}{\partial \ln \delta}\right|_{h(i)=\gamma(i)} &= \frac{\int_0^\delta \gamma(i)\frac{\partial x(i)}{\partial \ln \delta}di}{\int_0^\delta \gamma(i)x(i)di} - \frac{\partial \ln I}{\partial \ln \delta} \\
&= \frac{1}{I}\int_0^\delta \gamma(i)\frac{\partial x(i)}{\partial \ln \delta}di - \frac{\partial \ln I}{\partial \ln \delta} \\
&= \frac{1}{I}\int_0^\delta \frac{\partial \gamma(i)x(i)}{\partial \ln \delta}di - \frac{\partial \ln I}{\partial \ln \delta} \\
&= \int_0^\delta \frac{\partial \ln I}{\partial I}\frac{\partial \gamma(i)x(i)}{\partial \ln \delta}di - \frac{\partial \ln I}{\partial \ln \delta} \\
&= \frac{\partial \ln I}{\partial \ln \delta}\int_0^\delta \frac{\partial \gamma(i)x(i)}{\partial I}di - \frac{\partial \ln I}{\partial \ln \delta} = 0
\end{aligned}
$$

back

Appendix

$$
\begin{aligned}
\left.\frac{\partial \ln \mathrm{LP}}{\partial \ln \delta}\right|_{h(i)=1} = \frac{\partial \ln X}{\partial \ln \delta} - \frac{\partial \ln l}{\partial \ln \delta} &= \frac{\partial \ln \alpha(\delta)}{\partial \ln \delta} - \frac{\partial \ln \beta(\delta)}{\partial \ln \delta} \\
\text{(since } X = \int_0^\delta \frac{w\left[\gamma(\delta) - \gamma(i)\right]}{2b'(1-e)} di &= \frac{w}{2b'(1-e)} \delta \left(\gamma(\delta) - \mu'_\gamma\right) \\
&= \frac{w\alpha(\delta)}{2b'(1-e)}) \\
&= \frac{\delta \alpha_\delta}{\alpha(\delta)} - \frac{\delta \beta_\delta}{\beta(\delta)} \\
&= -\frac{\delta^2 \alpha_\delta \sigma_\gamma^2}{\alpha(\delta)\beta(\delta)} < 0
\end{aligned}
$$

back

Appendix

$$\left.\frac{\partial \ln \mathrm{LP}}{\partial \ln \delta}\right|_{h(i)=p(i)} = \frac{\int_0^\delta p(i)\frac{\partial x(i)}{\partial \ln \delta}di}{\int_0^\delta p(i)x(i)di} - \frac{\partial \ln I}{\partial \ln \delta}$$

$$\begin{aligned}
p(i)x(i) &= \frac{1}{2}\left(a' + w\gamma(i) - b'e(Y - X)\right)\frac{w}{2b'(1-e)}\left[\gamma(\delta) - \gamma(i)\right] \\
&= \frac{1}{2}\left(w\gamma(i) + w\gamma(\delta) + 2b'eX\right)\frac{w}{2b'(1-e)}\left[\gamma(\delta) - \gamma(i)\right] \\
&= w\left(\frac{1}{2}\left(\gamma(i) + \gamma(\delta)\right) + e\frac{\alpha(\delta)}{2(1-e)}\right)\frac{w}{2b'(1-e)}\left[\gamma(\delta) - \gamma(i)\right]
\end{aligned}$$

back

Appendix

$$X = \int_0^{\delta} \frac{a' - w\gamma(i) - b'e(X+Y)}{2b'(1-e)} di$$
$$= \frac{1}{2b'(1-e)} \int_0^{\delta} (a' - w\gamma(i) - b'e(1+km)X) di$$

$$X\left(1 + \frac{b'e\delta(1+km)}{2b'(1-e)}\right) = \frac{1}{2b'(1-e)} \int_0^{\delta} (a' - w\gamma(i)) di$$
$$X\left(\frac{2b'(1-e) + b'e\delta(1+km)}{2b'(1-e)}\right) = \frac{\delta}{2b'(1-e)} \left(a' - w\mu'_{\gamma}\right)$$

$$X = \frac{\delta}{b'(2(1-e) + e\delta(1+km))} \left(a' - w\mu'_{\gamma}\right)$$

back

Appendix - Industry Equilibrium Comparative Statics

$$\begin{bmatrix} \triangle_1 & 0 \\ e(1+km) & \frac{2(1-e)\delta\gamma_\delta}{\alpha(\delta)} \end{bmatrix} \begin{bmatrix} d\ln X \\ d\ln\delta \end{bmatrix} = \begin{bmatrix} \triangle_1 \\ e(1+km) \end{bmatrix} d\ln L$$

$$-ekm \begin{bmatrix} \delta \\ 1 \end{bmatrix} d\ln m + \begin{bmatrix} \triangle_0 \\ e \end{bmatrix} d\ln k - \begin{bmatrix} \delta\mu'_\gamma \\ \gamma(\delta) \end{bmatrix} \frac{2(1-e)}{\alpha(\delta)} d\ln w$$

$$d\ln X = d\ln L - \frac{e\delta km}{\triangle_1} d\ln m + \frac{\triangle_0}{\triangle_1} d\ln k - \frac{2(1-e)\delta\mu'_\gamma}{\triangle_1\ \alpha(\delta)} d\ln w$$

$$d\ln\delta = -\frac{e\delta km\alpha(\delta)}{\triangle_1\ \delta\alpha_\delta}(d\ln m + d\ln k) - \frac{2(1-e)\delta\mu'_\delta + \triangle_1\ \alpha(\delta)}{\triangle_1\ \delta\alpha_\delta} d\ln w$$

$$d\ln x(i) = d\ln L - \frac{ekm\alpha(\delta)}{\triangle_1\ [\gamma(\delta)-\gamma(i)]} d\ln m + \left[\frac{\triangle_0}{\triangle_1} + \left(1 - \frac{\triangle_0}{\triangle_1}\right)\right.$$

$$\left.\frac{\mu'_\gamma - \gamma(i)}{\gamma(\delta)-\gamma(i)}\right] d\ln k - \frac{2(1-e)\gamma(i) - e\delta(1+km)\left[\mu'_\gamma - \gamma(i)\right]}{\triangle_1\ [\gamma(\delta)-\gamma(i)]} d\ln w$$

back

Appendix - Industry Equilibrium Comparative Statics

$$
\begin{aligned}
0 &= \frac{\Delta_0}{\Delta_1} + \left(1 - \frac{\Delta_0}{\Delta_1}\right) \frac{\mu'_\gamma - \gamma(i)}{\gamma(\delta) - \gamma(i)} \\
(\gamma(i) - \gamma(\delta)) \frac{\Delta_0}{\Delta_1} &= \left(1 - \frac{\Delta_0}{\Delta_1}\right) (\mu'_\gamma - \gamma(i)) \\
\widetilde{\gamma}^{PE} = \gamma(i) &= \frac{\Delta_0}{\Delta_1}\gamma(\delta) + \left(1 - \frac{\Delta_0}{\Delta_1}\right) \mu'_\gamma
\end{aligned}
$$

back