# Gains from Openness with Heterogenous Firms

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# Overview

# Motivation

- International trade research has devoted a lot of attention on trade gains, less attention has been paid to measure gains. from FDI
- Arkolakis,derived gains from trade from the most commonly used quantitative trade models defined as $G_j = 1 - (\lambda_{jj})^{-\frac{1}{\varepsilon}}$
- The derivation is based on the observed share of a country's trade with itself, $\lambda_j$, and the elasticity of imports with respect to variable trade costs, $\varepsilon$
- In the first quarter of 2013, the magnitude of global FDI inflows and outflows were 357 and 353 billion US dollars respectively. [OECD (2013) FDI statistics report]
- Multinationals comprise a substantial majority of U.S. trade, roughly 90% of U.S. exports and imports (Bernard et al (2009))

# Research Question

- Are total gains from openness underestimated by omission of gains from FDI: To that effect how does the welfare measure of trade [a'la Arkolakis et al(2012)] change with FDI(gains from openess measure)
- **Result Preview** Define country $j$'s gains from international trade(openess) $G_j(C_j)$, as the absolute value of the % change in real income associated with moving from an observed equilibrium to autarky

| Gains from Trade | Current resuslt: Gains from Openness |
|---|---|
| $G_j = 1 - (\lambda_j)^{-\frac{1}{\varepsilon}}$ | $C_j = 1 - \left[\lambda_j^{ex-\frac{1}{\varepsilon}} + (\mathcal{R}_j^{\eta(\sigma-1)} - 1)^{\frac{(\sigma-1)-\varepsilon}{(\sigma-1)\varepsilon}} \lambda_j^{fdi-\frac{1}{\varepsilon}}\right]$ |

# Literature

## Proximity concentration trade off

- Brainard(1997) presents a simple theory to understand the trade-off between export and FDI.
- Markusen and Venables (2000)add factor endowment differences between countries to this simple model.
- Helpman, Melitz and Yeaple (2004) add firm heterogeneity

## Gains from Openness...?

- Eaton-Kortoum(2002), the only way of serving a foreign market is via exporting.
- Romondo(2008), variant of Eaton and Kortum, in which there are no trade flows.
- Ramondo Rodriguez-Clare(2009),Trade and multinational MP in an Eaton and Kortum model.
- Arkolakis et al(2012)New trade models same old gains with average sectoral elasticity.
- Ralph Ossa (2012) Average elasticity underestimates gains from trade, different sectoral elasticities more trade gains.

# Assumptions

- Following Helpman et al (2004), there are $N$ countries.
- Each country $i$ is endowed with $L_i$ units of labour with wage rate $w_i$.
- Each country comprises $H+1$ sectors, sector 0 produces a homogeneous good with 1 unit of labour per unit output,
- Sectors $h \geq 1$ produce differenciated products.
- The homogeneous good is freely traded with wage rate equal to 1 this ensures factor price equalisation as long as each and every country produces it.
- An exogenous fraction of income $\sum_h 1 - \beta_h$ is spent on the homogeneous sector and,
- $\beta_h$ is spent on the differentiated goods sector

# Preferences and Demand

- Preferences are a Cobb-Douglas aggregate of the homogeneous good sector and differentiated traded goods
- CES across a continuum of differentiated goods in $h = 1, ..., H$ sectors, (Krugman, 1980)

$$U = q_0^{1-\beta_h} \prod_{h=1}^{H} \left( \int_{\omega \in \Omega_h} q_h(\omega)^{\frac{\sigma_h - 1}{\sigma_h}} d\omega \right)^{\frac{\sigma_h}{\sigma_h - 1}\beta_h} \tag{1}$$

where $\sigma > 1$ is the elasticity of substitution ($\rho = \frac{\sigma-1}{\sigma}$)

- Utility maximization implies quantity demanded in county $j$ of good $\omega$ is

$$q_j(\omega) = \beta \frac{p_j(\omega)^{-\sigma}}{P_j^{1-\sigma}} Y_j \tag{2}$$

# Production

- Firms in country $i$ can enter the domestic market by paying fixed costs of entry $f_{E_i} > 0$ which is thereafter sunk.
- Expectation of future positive profits is the only motivation for firms to pay these sunk costs.

- **Exporting firm costs**

Exporting firms pay additional fixed ($f_{ij}^{ex} > 0=$ for each export destination. These marginal costs are given by,

$$C_{ij}^{ex} = \frac{w_i \tau_{ij}}{\varphi_{ij}} \tag{3}$$

# production continued...

- **FDI firm costs**

  FDI firms pay fixed costs of FDI defined as $f_{ij}^{fdi}$. FDI marginal costs are :

$$C_{ij}^{fdi} = \left(\frac{1}{\varphi_{ij}^{fdi}}\right) \left(\frac{w_j}{\eta}\right)^{\eta} \left(\frac{w_i \tau_{ij}}{1-\eta}\right)^{1-\eta} \quad (4)$$

- **Mode of Supply decisions**
- A firm will serve a foreign country if the operating profits are sufficient to cover fixed costs.
- A firm can choose to supply foreign markets via exports or set up FDI plants in foreign markets.

# production continued...

- If firm chooses to supply foreign market via export, profits are given by:

$$\pi_{ij}^{ex} = \beta P_j^{\sigma-1} \left( \frac{(\rho w_i \tau_{ij})^{1-\sigma}}{\varphi} \right)^{1-\sigma} \frac{Y_j}{\sigma} f_{ex} \tag{5}$$

- accesing foreign markets via FDI gives the following profits

$$\pi_{ij}^{fdi} = \beta P_j^{\sigma-1} \left( \frac{\rho w_j^{\eta} (w_i \eta_{ij})^{1-\eta}}{\varphi} \right)^{1-\sigma} \frac{Y_j}{\sigma} f_{fdi} \tag{6}$$

- **Cutoff Productivities**

  From zero profit condition, we get cutoff productivies for export and FDI

# production continued...

- **Export Productivity cutoff**

$$\tilde{\varphi}_{ij}^{ex} = \kappa \left( \frac{f_{ij}^{ex}}{Y_j} \right)^{\frac{1}{\sigma-1}} P_j^{-1} w_i \tau_{ij} \tag{7}$$

such that $\kappa = \left(\frac{\sigma}{\beta}\right)^{\frac{1}{\sigma-1}} \rho$

- **FDI Productivity cutoff**

$$\tilde{\varphi}_{ij}^{fdi} = \kappa \left( \frac{f_{ij}^{fdi} - f_{ij}^{ex}}{Y_j[(\mathcal{R}_{ij}\tau_{ij})^{\eta(\sigma-1)} - 1]} \right)^{\frac{1}{\sigma-1}} P_j^{-1} w_i \tau_{ij} \tag{8}$$

To ensure that the fdi cutoff productivity is higher than export productivity,i.e $\varphi_{ij}^{fdi} > \varphi_{ij}^{ex}$,assume that $f_{fdi}\mathcal{R}_{ij}^{\eta(\sigma-1)} > f_{ex}\tau^{\eta(\sigma-1)}$

# Entry and Exit

Timing

- Firms are identical prior to entry and must pay a xed investment cost $f_E$ to enter.
- Upon entry, firms draw their initial productivity level, $\varphi$ from a common distribution $g(\varphi) = k(\varphi_m in)^k \varphi^{-(k+1)}$ with positive support over $(0, \varphi_{max})$.and a continuous cumulative distribution $G(\varphi) = 1 - \left(\frac{\varphi_{min_i}}{\varphi}\right)^{\kappa}$
- A firm drawing a low productivity $\varphi$ may decide to immediately exit and not produce.
- Produtivities are distributed Pareto
- Firms face a constant probability of a bad shock in every period that would force them to $\delta$ exit.

# Entry and Exit continued

- **Pareto Distribution**
- Good approximation of the upper tail of distribution of firm sizes (Simon and Bonini,1958).
- key feature 1: when truncated the random variable retains a Pareto distribution with the same shape parameter k.
- If entry is subject to an endogenous productivity cutoff, the distribution of the technologies that make the cut remains Pareto
- key feature 2: Pareto distributed random variable power functions are themselves Pareto distributed.
- Individual prices have a Pareto distribution, with a constant elasticity of demand, so do sales, hence firm size and variable profits are Pareto distributed.

# Entry and Exit Continued

- Probability of entry in the home market, exporting (conditional on successful entry)and into FDI are given by

$$\theta_{iD} = 1 - G(\varphi_i^*) \tag{9}$$

$$\theta_{ex} = \frac{G(\varphi_{fdi}^*) - G(\varphi_{ex}^*}{1 - G(\varphi_i^*)} \tag{10}$$

$$\theta_{fdi} = \frac{1 - G(\varphi_{fdi}^*)}{1 - G(\varphi_i^*)} \tag{11}$$

# Entry and Exit continued...

- **Free Entry Condition** Expectation of future profits must be equal to the fixed costs or that the probability of successful entry multiplied by average profits conditional on successful entry equal fixed costs

$$\overline{v}(\varphi) = \sum_{t=0}^{\infty}(1-\delta)\overline{\pi} = \frac{[1-G(\varphi^*)]\overline{\pi}}{\delta} - f_e \tag{12}$$

Where the average revenues and profits across all domestic firms earned from both domestic, export and FDI revenues are given as:

$$\overline{r} = \pi_{iD}(\tilde{\varphi}) + n\theta_{ex} r_{ex}(\tilde{\varphi}) + n\theta_{fdi} r_{fdi}(\tilde{\varphi}) \tag{13}$$

# Entry and Exit continued...

- Taking into account the zero profit condition the previous equation becomes,

$$\bar{r} = \sigma(\bar{\pi} + f_{iD} + n\theta_{ex} f_{ex} + n\theta_{fdi} f_{fdi}) \quad (14)$$

$$\bar{\pi} = \pi_{iD}(\tilde{\varphi}) + n\theta_{ex}\pi_{ex}(\tilde{\varphi}) + n\theta_{fdi}\pi_{fdi}(\tilde{\varphi}) \quad (15)$$

- These average Profits pin down the equilibrium mass of incumbent firms given as

$$M = \frac{R}{\bar{r}} = \frac{L}{\sigma(\bar{\pi} + f_e + n\theta_{ex} f_{ex} + n\theta_{fdi} f_{fdi})} \quad (16)$$

# Aggregation

- Let $M$ be the equilibrium mass of incumbent firms in any country,
- Mass of firms that enter foreign country via exports and FDI $M_{ex} = \theta_{ex} M$ and $M_{fdi} = \theta_{fdi} M$
- Total mass of varieties available to consumers in each country is given by the total mass of firms competing in the country,

$$M = M + nM_{ex} + nM_{fdi} \tag{17}$$

- Weighted productivity average

$$\widehat{\varphi} = \frac{1}{M}\left( M\widehat{\varphi}_{iD}^{\sigma-1} + nM_{ex}\tau^{1-\sigma}\widehat{\varphi}_{ex}^{\sigma-1} + nM_{fdi}\tau^{(1-\sigma)(1-\eta)}\widehat{\varphi}_{fdi}^{\sigma-1} \right)^{\frac{1}{\sigma-1}} \tag{18}$$

$P = M^{\frac{1}{1-\sigma}} p(\widehat{\varphi}),\ W = M^{\frac{1}{\sigma-1}} \rho\widehat{\varphi}$

# Aggregation continued...

- **Trade only Price Index**

$$P_j^{ex} = \frac{G(\varphi^*_{fdi}) - G(\varphi^*_{ex})}{1 - G(\varphi^*_i)} N_{ij}^{ex} \left( \int_{\varphi_{ij}^{ex}}^{\varphi_{ij}^{fdi}} \varphi^{\sigma-1} (\frac{\sigma}{\sigma - 1} w_{ij} \tau_{ij})^{1-\sigma} dG(\varphi_{ij}) \right)^{\frac{1}{1-\sigma}} \quad (19)$$

Evaluating the integral and substituting for cutoff productivities

$$P_j^{-k} = M_{ij} \frac{k}{k - \sigma + 1} \left( \frac{1}{Y_j} \right)^{1 - \frac{k}{\sigma - 1}} \varphi_{min}^k \left( \frac{\sigma}{\sigma - 1} w_i \tau_{ij} \right)^{-k} (\sigma w_j f_{ij}^{ex}))^{1 - \frac{k}{\sigma - 1}} \quad (20)$$

# Aggregation continued...

- **FDI only price Index**

$$P_{ij}^{fdi} = \frac{1 - G(\varphi_{fdi}^*)}{1 - G(\varphi_i^*)} M_{ij}^{fdi} \left( \int_{\varphi_{ij}^{fdi}}^{\infty} \varphi^{\sigma-1} (\frac{\sigma}{\sigma - 1})^{1-\sigma} [(\Re_{ij}\tau_{ij})^{\eta(\sigma-1)} - 1](w_{ij}\tau_{ij})^{1-\sigma} dG(\varphi_{ij}) \right)^{\frac{1}{1-\sigma}} \quad (21)$$

Evaluating the integral and substituting for cutoff productivities

$$P_j^{-k} = M_{ij} \frac{k}{k - \sigma + 1} \left( \frac{1}{Y_j} \right)^{1 - \frac{k}{\sigma - 1}} \varphi_{min}^k \left( \frac{\sigma}{\sigma - 1} w_i \tau_{ij} \right)^{-k} \left( \sigma w_j ( \frac{f_{fdi} - f_{ex}}{(\Re\tau_{ij})^{\eta(\sigma-1)} - 1}) \right)^{1 - \frac{k}{\sigma - 1}} \quad (22)$$

# Aggregation continued...

- **Aggregate price Index price Index**

$$P_{ij}^{1-\sigma} = \frac{G(\varphi_{fdi}^*) - G(\varphi_{ex}^*)}{1 - G(\varphi_i^*)} M_{ij}^{ex} \left( \int_{\varphi_{ij}^{ex}}^{\infty} \varphi^{\sigma-1} (\frac{\sigma}{\sigma-1} w_{ij}\tau_{ij})^{1-\sigma} dG(\varphi_{ij}) \right)$$
$$+ \frac{1 - G(\varphi_{fdi}^*)}{1 - G(\varphi_i^*)} N_{ij}^{fdi} \left( \int_{\varphi_{ij}^{fdi}}^{\infty} \varphi^{\sigma-1} (\frac{\sigma}{\sigma-1})^{1-\sigma} [(\Re_{ij}\tau_{ij})^{\eta(\sigma-1)} - 1](w_{ij}\tau_{ij})^{1-\sigma} dG(\varphi_{ij}) \right)$$

Evaluating the Integral and substituting for the cut off productivities

$$P_j^{-k} = M_{ij} \frac{k}{k-\sigma+1} \left(\frac{1}{Y_j}\right)^{1-\frac{k}{\sigma-1}} \left(\frac{\sigma}{\sigma-1}(w_i\tau_{ij})\right)^{-k} (\sigma w_j)^{1-\frac{k}{\sigma-1}}$$
$$\left( f(_{ij}^{ex})^{1-\frac{k}{\sigma-1}} + (\frac{f_{fdi} - f_{ex}}{(\Re\tau_{ij})^{\eta(\sigma-1)} - 1})^{1-\frac{k}{\sigma-1}} \right)$$

# Aggregation continued...

- **Trade Sales**

$$X_{ij}^{ex} = \frac{G(\varphi_{fdi}^{*}) - G(\varphi_{ex}^{*})}{1 - G(\varphi_{i}^{*})} M_{ij}^{ex} \left( \int_{\varphi_{ij}^{ex}}^{\infty} \varphi^{\sigma-1} (\frac{\sigma}{\sigma-1} w_{ij}\tau_{ij})^{1-\sigma} \beta \frac{Y_j}{P_j^{1-\sigma}} dG(\varphi_{ij}^{ex}) \right) \quad (23)$$

Evaluating the integral and substituting for productivity cutoffs we get

$$X_{ij}^{ex} = \underbrace{\left(\frac{\varphi_{min}}{\varphi_{ex}}\right)^{k} M_{ij}^{ex}}_{\text{extensive}} \underbrace{\left(\frac{\sigma k}{k-\sigma+1}\right) f_{ex} w_j}_{\text{intensive}} \quad (24)$$

The effect of trade costs on export sales also yields the extensive and intensive margins as

$$\frac{dlnX_{ij}^{ex}}{dln\tau_{ij}} = -\underbrace{(\sigma-1)}_{\text{intensive}} - \underbrace{(k-\sigma+1)}_{\text{extensive}} \quad (25)$$

# Aggregation continued...

- **FDI Sales**

$$X_{ij}^{fdi} = \frac{1-G(\varphi_{fdi}^{*})}{1-G(\varphi_{i}^{*})} M_{ij}^{fdi} \int_{\varphi_{ij}^{fdi}}^{\infty} \varphi^{\sigma-1} (\frac{\sigma}{\sigma-1})^{1-\sigma} [(\Re_{ij}\tau_{ij})^{\eta(\sigma-1)} - 1](w_{ij}\tau_{ij})^{1-\sigma} \beta \frac{Y_j}{P_j^{1-\sigma}} dG(\varphi_{ij}^{fdi})$$

$$X_{ij}^{fdi} = \underbrace{\left(\frac{\varphi_{min}}{\varphi_{fdi}}\right)^{k} M_{ij}^{fdi}}_{\text{extensive}} \underbrace{\left(\frac{\sigma k}{k-\sigma+1}\right) w_j (f_{fdi} - f_{ex})}_{\text{intensive}} \tag{26}$$

# Aggregation continued...

Effect of an increase in variable trade barriers on total affiliate sales can be decomposed into intensive and extensive margin

$$\frac{dlnX_{ij}^{fdi}}{dln\tau_{ij}} = -\underbrace{(1-\eta)(\sigma-1)}_{\text{intensive}} - \underbrace{(k-\sigma+1)\chi_{fdi}}_{\text{extensive}} \tag{27}$$

Where $\chi_{fdi}$ is defined as the elasticity of FDI cutoff to variable trade barriers.

$$\chi_{fdi} = \frac{(\mathcal{R}_{ij}\tau_{ij})^{\eta(\sigma-1)}(\eta-1)-1}{(\mathcal{R}_{ij}\tau_{ij})^{\eta(\sigma-1)}-1} \tag{28}$$

# Definition of gains from openness

- *Gains from openess equals the (absolute value of) the percentage change in real income associated with moving one country from the current, observed trade and fdi equilibrium to a counterfactual equilibrium*
- *We are interested in the effects of trade and FDI on the welfare measure.*
- *Welfare is given by the per capita value of real income accruing to consumers:*

$$W_j = \frac{Y_j}{L_j P_j} = \frac{w_j^{ex}}{P_j^{ex}} + \frac{w_j^{fdi}}{P_j^{fdi}} \tag{29}$$

- *welfare depends on real labour income derived from export and foreign multinational affiliates*

- ACR express country $j$ welfare as a function of the share of expenditure that falls on domestically produced goods(which is equal to 1 minus the import penetration ratio. This share, $\lambda_j$ under autarky is 1,therefore total size of gains from openness will be equal to $1 - \lambda_j$
- changes in $W_j$ can be inferred from changes in bilateral trade and income expenditures alone.

# Expenditure shares

- **Trade expediture share**

$$\lambda_{ij}^{ex} = \frac{X_{ij}^{ex}}{\sum_v X_{vj}} = \frac{\left(\frac{\varphi_{ii}^*}{\varphi_{ij}^*}\right)^k N_i w_i f_{ij_{ex}} \frac{\sigma k}{k-\sigma+1}}{\sum_v \left(\frac{\varphi_{ii}^*}{\varphi_{vj}^*}\right)^k N_i w_i f_{vj_{ex}} \frac{\sigma k}{k-\sigma+1}} \tag{30}$$

Insert the equilibrium mass of firms and export cutoff productivty:

$$\lambda_{ij}^{ex} = \frac{(L_i/f_{iE})\varphi_{min_i}^k w_i^{-\left(\frac{k\sigma-(\sigma-1)}{\sigma-1}\right)} (\tau_{ij})^{-k} f_{ij_{ex}}^{1-\frac{k}{\sigma-1}}}{\sum_v (L_{vj}/f_{vE})\varphi_{min_v}^k w_v^{-\left(\frac{k\sigma-(\sigma-1)}{\sigma-1}\right)} (\tau_{vj})^{-k} f_{vj_{ex}}^{1-\frac{k}{\sigma-1}}} \tag{31}$$

# Expenditure shares(Continued)

- **FDI expediture share**

$$\lambda_{ij}^{fdi} = \frac{X_{ij}^{fdi}}{\sum_v X_{vj}} = \frac{\left(\frac{\varphi_{ii}^*}{\varphi_{ij}^*}\right)^k N_i w_i (f_{ij_{fdi}} - f_{ij_{ex}}) \frac{\sigma k}{k-\sigma+1}}{\sum_v \left(\frac{\varphi_{ii}^*}{\varphi_{vj}^*}\right)^k N_i w_i (f_{vj_{fdi}} - f_{vj_{ex}}) \frac{\sigma k}{k-\sigma+1}} \tag{32}$$

Insert the equilibrium mass of firms and FDI cutoff productivty:

$$\lambda_{ij}^{fdi} = \frac{(L_i/f_{iE})\varphi_{min_i}^k w_i^{-\left(\frac{k\sigma-(\sigma-1)}{\sigma-1}\right)} (\tau_{ij})^{-k} f_{ij_{ex}}^{1-\frac{k}{\sigma-1}} (\Re_{vj}\tau_{ij})^{\eta(\sigma-1)} - 1)^{1-\frac{k}{\sigma-1}}}{\sum_v (L_{vj}/f_{vE})\varphi_{min_v}^k w_v^{-\left(\frac{k\sigma-(\sigma-1)}{\sigma-1}\right)} (\tau_{vj})^{-k} f_{vj_{ex}}^{1-\frac{k}{\sigma-1}} (\Re_{vj}\tau_{vj})^{\eta(\sigma-1)} - 1)^{1-\frac{k}{\sigma-1}}} \tag{33}$$

# Welfare derivation

- Knowing a country's domestic share of trade and FDI and shape parameter of the productivity distribution $k$ is sufficient to determine welfare gains from openness.
- To achieve this, write the export(FDI) price index as a function of country $j$'s share of trade(FDI) with itself, its wage, labour allocation and parameters.
- **Welfare derived from Trade**

$$\left(P_j^{ex}\right)^{-k} = \sum_i (L_i/f_E)\varphi_{min_i}^k w_i^{-k}(\tau_{ij})^{-k} f_{ex}^{1-\frac{k}{\sigma-1}} \left(\frac{w_j^{ex}}{Y_j^{ex}}\right)^{1-\frac{k}{\sigma-1}} \left(\frac{\sigma}{\sigma-1}\right)^{-k} \sigma^{-\frac{k}{\sigma-1}} \frac{\sigma-1}{k-\sigma+} \tag{34}$$

We use the expression of country $j$'s expenditure with itself, $\lambda_{jj}^{ex}$ and the expression for income derived from export sales, $Y_j^{ex} = w_j^{ex} L_j^{ex}$

# Welfare derivation (Continued)

$$W_j^{ex} = \left(\frac{w_j^{ex}}{P_j^{ex}}\right) = \left[\left(\frac{\sigma}{\sigma-1}\right)^{-k} \sigma^{-\frac{k}{\sigma-1}} \frac{\sigma-1}{k-\sigma+1} \varphi_{min_j}^{k} f_{ex}^{1-\frac{k}{\sigma-1}} f_E\right]^{\frac{1}{k}} (L_j^{ex})^{\frac{1}{\sigma-1}} (\lambda_{jj}^{ex})^{-\frac{1}{k}} \tag{35}$$

We summarize the welfare gains from trade, measured as the welfare ratio between the open and closed economies(the domestic trade shares in the closed economy are xed at $\lambda^{closed} = 1$):

$$\widehat{W}_j^{ex} = \frac{W_j^{open}}{W_j^{closed}} = \widehat{\lambda}_{j_{ex}}^{-\frac{1}{k}} \tag{36}$$

The change in real income derived from trade associated with a change in the country's expenditures on domestic goods.

- **Welfare derived from FDI**

$$\left(P_j^{fdi}\right)^{-k} = \sum_i (L_i/f_{iE})\varphi_{min_i}^k w_i^{-k}(\tau_{ij})^{-k}[(\mathcal{R}_{ij}\tau_{ij})^{\mu(\sigma-1)}-1]^{-\left(1-\frac{k}{\sigma-1}\right)}$$
$$(f_{fdi}-f_{ex})^{1-\frac{k}{\sigma-1}}\left(\frac{w_j^{fdi}}{Y_j^{fdi}}\right)^{1-\frac{k}{\sigma-1}}\left(\frac{\sigma}{\sigma-1}\right)^{-k}\sigma^{-\frac{k}{\sigma-1}}$$
$$\frac{\sigma-1}{k-\sigma+1}$$

Now we use the expression of country $j$'s expenditure with itself, $\lambda_{jj}^{fdi}$ and the expression for income derived from FDI, $Y_j^{fdi} = w_j^{fdi}L_j^{fdi}$

$$W_j^{fdi} = \left(\frac{w_j^{fdi}}{P_j^{fdi}}\right) = \left[\left(\frac{\sigma}{\sigma-1}\right)^{-k}\sigma^{-\frac{k}{\sigma-1}}\frac{\sigma-1}{k-\sigma+1}\varphi_{min_j}^k(f_{fdi}-f_{ex})^{1-\frac{k}{\sigma-1}}f_{jE}^{-1}\right]^{\frac{1}{k}}$$
$$(L_j^{fdi})^{\frac{1}{\sigma-1}}[\mathcal{R}_j^{\eta(\sigma-1)}-1]^{\frac{(\sigma-1)-k}{(\sigma-1)k}}\left(\lambda_{jj}^{fdi}\right)^{-\frac{1}{k}}$$

$$\widehat{W}_j^{fdi} = [\hat{\mathcal{R}}_{jj}^{\eta(\sigma-1)}-1]^{\frac{(\sigma-1)-k}{(\sigma-1)k}}\widehat{\lambda}_{j^{fdi}}^{-\frac{1}{k}} \qquad (37)$$

- **Gains from Openness**

$$\begin{aligned}\widehat{W}_j &= \widehat{W}_j^{ex} + \widehat{W}_j^{fdi} \\ &= \lambda_j^{ex^{-\frac{1}{\varepsilon}}} + (\mathcal{R}_j^{\eta(\sigma-1)} - 1)^{\frac{(\sigma-1)-\varepsilon}{(\sigma-1)\varepsilon}} \lambda_j^{fdi^{-\frac{1}{\varepsilon}}}\end{aligned}$$

**Country $j$'s gains from openness $C_j$ is the absolute value of the percentage change in real income associated with moving to the counterfactual equilibrium:**

$$C_j = 1 - \left[\lambda_j^{ex^{-\frac{1}{\varepsilon}}} + (\mathcal{R}_j^{\eta(\sigma-1)} - 1)^{\frac{(\sigma-1)-\varepsilon}{(\sigma-1)\varepsilon}} \lambda_j^{fdi^{-\frac{1}{\varepsilon}}}\right] \tag{38}$$

# Summary

- We have revisited the welfare gains from trade in the presence of FDI.
- Gains from openness are underestimated by a trade only model: FDI provides an addition source of gains to trade gains.

**Thank you**