Gains from Openness with Heterogeneous Firms

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Abstract

International trade research has used quantitative trade models to quantify gains from trade. Gains from trade derived by [1] were shown to be the same in all quantitative trade models under both perfect competition models [2] and [6] and monopolistic competition [16]. The derivation is based on the observed share of a country's trade with itself(share of expenditure on domestic goods), λ_j, and the elasticity of aggregate trade costs, ε. However, it is rather puzzling that despite significant FDI transactions at regional and global level, less attention has been devoted to the additional gains from FDI in a heterogenous firm environment. The repercussions of such an omission may lead to underestimation of total gains from openness(both trade and FDI). Therefore main contribution of our work is to demonstrate that there are additional welfare gains from FDI.

Keywords:Exports, Free Trade, International Trade, Trade, Welfare,Foreign Direct Investment,Multinational Firm

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1 Introduction

Despite substantial volumes of FDI inflows and outflows, it is only until recent, that international trade research has paid attention to measure gains from FDI, much of the attention has been devoted to trade gains. According to the OECD (2013) foreign direct investment(FDI) statistics report, in the first quarter of 2013, the magnitude of global FDI inflows and outflows were 357 and 353 billion US dollars respectively. Moreover, [17] stated that worldwide sales of mutinational companies are in the order of twice that of total exports. Based on this revelation, the objective of this study is to measure gains from Trade and FDI. Where current research considers gains from FDI and Trade, their approach differs from a heterogeneous productivity framework of [16] and [9]. For example, [17] estimated gains from openness based on Eaton and Kortum framework.

Gains from FDI include amongst others (a) gains from resource transfer; [7], [13], [14], and [18], asserted that FDI brings physical capital, new technology,increased competition and improved methods of conducting business . These resources are transfered into domestic firms leading to increased average productivity, increased product and process innovations,(b) employment gains; FDI has the potetial to increase emplyment in the local economy which leads to increased spending and incresed multiplier effects in the domestic economy, (c) balance of payments effects;gains from the balance of payments effects are derived from improvement in the capital account due to the inflows of new capital into the host country and improvements in the current account balance because of possible decline in imports of goods and service (d) technology spillovers; [10] noted that technological spillover benefits are derived from adopting the product, process and organizational innovations initiated by the mutinationl firm. It is worthwhile to mention at this point that, in our present study we only consider the change in per capita value of real income accruing to consumers as the measure of gains from FDI and trade.

In our analysis, gains from openness equals the (absolute value of) the percentage change in real income associated with moving one country from the current, observed trade and fdi equilibrium to a counterfactual equilibrium. Gains from trade were derived by [1] as $G_j = 1 - (\lambda_j)^{-\frac{1}{\varepsilon}}$ two static measures contributed to the welfare predictions;(i) share of expenditure on domestic goods and (ii)an elasticity of imports with respect to variable trade costs, (trade elasticity). In this study we find that there are additional gains from FDI which is facilitated by traded intermediate inputs. Consequently total gains from trade and FDI(gains from openness) are; $C_j = 1 - \left[\lambda_j^{ex-\frac{1}{\varepsilon}} + (\mathcal{R}_j^{\mu(\sigma-1)} - 1)^{\frac{(\sigma-1)-\varepsilon}{(\sigma-1)\varepsilon}} \lambda_j^{fdi-\frac{1}{\varepsilon}}\right]$. Where σ is the elasticity of substitution, μ is the ratio between wages in the destination country where a foreign affiliate is set up and intermediate input costs. Provided there are positive marginal cost savings $([\mathcal{R}_j^{\mu(\sigma-1)} - 1] > 0$. Therefore gains from openess (Trade and FDI) are higher than gains in a trade only model as shown by [1]. However when the marginal cost savings are zero, then gains from, FDI are zero, and we get only the gains from trade component.

The analysis of this study will be based on the model developed by [9] who extends the Melitz model by incorporating FDI and shows that firms sort into export and FDI based on heterogeneity on their productivity levels.They predict that only firms that are productive enough to cover the fixed cost of exporting can export. Since the fixed cost of FDI is larger than that of exporting, firms that conduct FDI must be more productive than firms that only export. However, we consider the use of intermediate goods in foreign affiliates production plants shipped fromcountry of origin of the multinational firm. This is consistent with [3] as they found out that multinationals comprise a substantial majority of U.S. trade, roughly 90% of U.S. exports and imports. This will allow us to examine the relationship between trade and FDI. This approach is closely related to [4], [15], [9] who asserted that trade and FDI are complementary, i.e., a rise in trade costs leads to a replacement of exports by FDI production in destination countries.

A closer model to our work is provided by [12], where foreign affiliate production used intermediate inputs in production shipped from the firms country of origin they found out that trade facilitates FDI in our and FDI boosts trade. A quantifiable multi-country general equilibrium model developed by [20] showed that gains from multinational production are much smaller due to fixed costs of establishing foreign plants. The author points out that these gains may increase significantly if free entry is taken into account. Therefore we allow for free firm entry in our model.

The rest of this paper is organised as follows, Section [2] presents a theoretical model, describing the assumptions, technology and cost structure of domestic, export and FDI firms, preferences and demand, and finally the firm decision problem. Section [3], shows aggregation and equilibrium of the model. Lastly section [4] gives welfare predictions derived from the model.

2 Model

Following [9], there are N countries. Each country i is endowed with L_i units of labour with wage rate w_i. Each country comprises 2 sectors, sector 0 produces a homogeneous good with 1 unit of labour per unit output, H sectors produce differenciated products. We assume that the homogeneous good is freely trade with wage rate equal to 1 and it is produced under a constant returns to scale which will ensure factor price equalisation as long as each and every country produces it. An exogenous fraction of income $\sum_h 1-\beta_h$ is spent on the homogeneous sector and β_h is spent on the differentiated goods sector.

2.1 Preferences and Demand

Preferences are a Cobb-Douglas aggregate of the homogeneous good sector and differentiated traded goods and CES across a continuum of differentiated goods in $h = 1, ..., H$ sectors.

$$U = q_0^{1-\beta_0} \prod_{h=1}^{H} \left(\int_{\omega_h} q_h(w)^{\frac{\sigma_h - 1}{\sigma_h}} \right)^{\frac{\sigma_h}{\sigma_h - 1}\beta_h} \tag{1}$$

The elasticity of substitution $\sigma > 1$ between varieties and within the firm is the same, hence we use the standard optimal pricing rule. We proceed in the same spirit as [9], we drop the sectoral index h in light of the view that all sectoral variables refers to a particular sector h that produces differentiated products. The consumer's problem is to maximize their utility subject to a budget constraint. This yields the following export and FDI demand functions for differentiated products in sector h respectively.

$$q_{ij}(\varphi) = \beta \frac{p_{ij}(\varphi)^{-\sigma}}{P_j^{1-\sigma}} Y_j \tag{2}$$

Where P_j is the ideal price index in country j, and p_{ij} is the price charged on goods.

With monopolistic competition and Dixit-Stiglitz preferences, the price charged is a constant mark-up $\frac{\sigma}{\sigma-1} = \rho$ of the costs of production, therefore we have

$$p_{ex}(\varphi) = \rho \frac{w_i \tau_{in}}{(\varphi)} \tag{3}$$

$$p_{fdi}(\varphi) = \rho \frac{(w_i)^{\mu}(w_i \tau_{ij})^{1-\mu}}{(\varphi)} \tag{4}$$

The price index P_j^h of the aggregate bundle of exports and FDI is the ideal price index of the industry h in country j

$$(P_j)^{1-\sigma} = \left(\int_{\varphi_{ij}^{ex}}^{\varphi_{ij}^{fdi}} (p_{ex})^{1-\sigma} dG(\varphi) + \int_{\varphi_{ij}^{fdi}}^{\infty} (p_{fdi})^{1-\sigma} dG(\varphi) \right) \tag{5}$$

The above aggregate price index can be split into price index for imported goods and price index for foreign affiliate goods as follows,(dropping all the sector sub scripts),

$$(P_j^{ex})^{1-\sigma} = M_{ij}^{ex} \left(\int_{\varphi_{ij}^{ex}}^{\varphi_{ij}^{fdi}} (p_{ex})^{1-\sigma} dG(\varphi) \right) = \left(N_{ij}^{ex}\right)^{\frac{1}{1-\sigma}} p_{ex}(\varphi_{ij}^{ex}) \tag{6}$$

$$(P_j^{fdi})^{1-\sigma} = M_{ij}^{fdi} \left(\int_{\varphi_{ij}^{fdi}}^{\infty} (p_{fdi})^{1-\sigma_h} dG(\varphi) \right) = \left(N_{ij}^{fdi}\right)^{\frac{1}{1-\sigma}} p_{fdi}(\varphi_{fdi}) \tag{7}$$

In both cases, the price index is a multiple of the distribution of lowest prices and mass of varieties M_{ij}^{ex} and M_{ij}^{fdi} for exports and fdi respectively. It is decreasing in average productivity cutoffs and number of varieties available, and increasing in wages, transportation costs. It then follows that the aggregate price index(9) can be written as the sum of equations (6) and (7) as:

$$(P_j)^{1-\sigma} = \left(M_{ij}^{ex}\right)^{\frac{1}{1-\sigma}} p_{ij}^{ex}(\varphi_{ij}^{ex}) + \left(M_{ij}^{fdi}\right)^{\frac{1}{1-\sigma}} p_{ij}^{fdi}(\varphi_{ij}^{ex}) \tag{8}$$

The aggregate price index is decreasing in cutoff productivities and total number of varieties provided by multinational production and imports. Since both exports of final and intermediate goods are subject to iceberg transportation costs, the aggregate price index in an increasing function of transportation costs, and wages paid to labour in foreign affiliate plant and country of origin.

2.2 Technology and Cost Structure

2.2.1 Domestic Firm Costs

Firms in country i can enter the domestic market by paying fixed costs of entry $f_{E_i} > 0$(measured in units of labour) which is thereafter sunk. Expectation of future positive profits is the only motivation for firms to sink these investment costs. After observing the productivity parameter φ from the distribution $G(\varphi)$.the A firm may start production and pay additional fixed costs $f_{D_i} > 0$ with marginal costs

$$C = \frac{w_i}{\varphi} \tag{9}$$

Depending on the level of their productivity, domestic firms have two alternatives of serving foreign markets, either through exports or setting up a foreign affiliate in the destination country. Proximity concentration trade off provides an impetus for decisions on the former and the latter.

2.2.2 Exporting Firm Costs

If φ is sufficiently low, a firm may decide to start exporting and pays additional fixed costs or each export destination. These fixed costs are given by $f_{ex} > 0$. The marginal cost for an exporting firm from country i to export destination j is given by

$$C_{ij}^{ex} = \frac{w_i \tau_{ij}}{\varphi} \tag{10}$$

Exports are subject to melting iceberg transportation costs $\tau_{ij} > 1$, i.e τ_{ij} units have to be shipped from country i for one unit of the good to arrive in country j. As in [16] the marginal costs in equation (10) of an exporting firm increase as the transportation and labour costs increase and they decrease with a rise in firm productivity.

2.2.3 FDI Firm Costs

If a firm originating from country i chooses to set up a production plant in a foreign country j it incurs fixed costs of FDI defined as f_{fdi}. These fixed costs envisage costs of setting up or acquiring a physical structure, marketing and information,etc.Following [11] we introduce intra-firm trade into FDI to be consistent with the facts that the relationship between export and outward FDI is complementary. However our analysis does not consider any sourcing strategies for intermediate goods as in the case of [8]. A thorough illustration of technological trade off between intermediate good production in the country of origin and in the destination country is provided by [12].

Foreign affiliates from country i in country j use intermediate inputs from country i, these intermediate goods are subject to melting iceberg transportation costs τ_{ij}. Therefore the cost function is of cobb-douglas form given by

$$C_{ij}^{fdi} = \left(\frac{1}{\varphi}\right)\left(\frac{w_j}{\mu}\right)^{\mu}\left(\frac{w_i\tau_{ij}}{1-\mu}\right)^{1-\mu} \tag{11}$$

On the right hand side of equation (11) the first term indicates the inverse relationship between the costs and productivity level in the same spirit as the discussion under exporting firm costs. The second term shows the wage rate in the destination country j and the third term is the cost of intermediate input used in the foreign affiliate production.Note that when $\mu = 1$ equation (11) yield identical marginal costs to those in the paper of export versus FDI with heterogenous firms by [9]. From (11), an increase in the size of trade costs between the affiliate and country of origin results in an increase in marginal trade costs of the affiliate.

$0 \leq \mu < 1$, is the ratio of foreign plant labour cost tointermidiate iputs used produce goods in foreign affiliates. This cost function derives from a cobb-douglas production function with a cost share μ for labour and $1-\mu$ for intermediate input. This can be represented as

$$Z^{fdi} = \varphi\left(\frac{L}{\mu}\right)^{\mu}\left(\frac{Q_j^{input}}{1-\mu}\right)^{1-\mu} \tag{12}$$

Export and FDI sales are therefore given by the following equation:

$$X_{ij_v}(\varphi_{ij}^{v}) = \beta\left(\frac{p_{ij_v}(\varphi)}{P_j}\right)^{1-\sigma} Y_j \tag{13}$$

$v = [ex, fdi]$

2.3 Firms Decision Problem and Cut-off Costs and Productivities

A firm from country i will serve a foreign country j if the operating profits are sufficient to cover costs of entry(fixed costs). We get exporting and fdi firm profits respectively as:

$$\pi_{ij}^{ex} = \beta P_j^{\sigma-1} \left(\frac{(\rho w_i \tau_{ij})^{1-\sigma}}{\varphi} \right)^{1-\sigma} \frac{Y_j}{\sigma} f_{ex} \tag{14}$$

$$\pi_{ij}^{fdi} = \beta P_j^{\sigma-1} \left(\frac{\rho w_j^{\mu} (w_i \tau_{ij})^{1-\mu}}{\varphi} \right)^{1-\sigma} \frac{Y_j}{\sigma} f_{fdi} \tag{15}$$

Firms will serve foreign markets through exports or FDI if the operating profits are greater than fixed costs. Therefore the zero profit condition for exporting and fdi firms is given by the following equations respectively:

$$\pi_{ij}^{ex} = 0 \Longrightarrow \beta P_j^{\sigma-1} \left(\frac{(\rho w_i \tau_{ij})^{1-\sigma}}{\varphi} \right)^{1-\sigma} \frac{Y_j}{\sigma} = f_{ex} \tag{16}$$

$$\pi_{ij}^{fdi} = 0 \Longrightarrow \beta P_j^{\sigma-1} \left(\frac{\rho w_j^{\mu} (w_i \tau_{ij})^{1-\mu}}{\varphi} \right)^{1-\sigma} \frac{Y_j}{\sigma} = f_{fdi} \tag{17}$$

2.3.1 Cut-off Productivities

1. **Export Productivity Cut Off**

   We revert to the zero profit condition for exporting forms and substitute export firm costs from equation(10). We find that the productivity cut-off for exporting firms is given by;

   $$\varphi_{ex}^{*} = \kappa \left(\frac{f_{ex}}{Y_j} \right)^{\frac{1}{\sigma-1}} P_j^{-1} w_i \tau_{ij} \tag{18}$$

   where $\kappa = \left(\frac{\sigma}{\beta}\right)^{\frac{1}{\sigma-1}} \rho$. Firms with an export productivity cut-off in equation (18) have just break even. Therefore firms with a higher productivity than export productivity cut-off in equation (18) expect to make positive profits. This result is consistent with [9] and with [16]

2. **FDI Productivity Cut-off**

   This is the productivity cutoff level where the firm is indifferent between Export and FDI. Moreover, if profits from having multinational foreign affiliates are greater than profits accrued from accessing foreign markets through exports, then FDI is the preferred choice. This is the same argument provided in [9].

$$\varphi^*_{fdi} = \kappa \left(\frac{f_{fdi} - f_{ex}}{Y_j[(\mathcal{R}_{ij}\tau_{ij})^{\mu(\sigma-1)} - 1]} \right)^{\frac{1}{\sigma-1}} P_j^{-1} w_i \tau_{ij} \tag{19}$$

Firms with a productivities between export and FDI cut-off productivities, i.e between (18) and (19) will serve the foreign market through exports only. Firms with a productivity cut-off given by (19) have just break even and make zero profits if they access the foreign market via FDI, However all firms with a productivity above the FDI productivity cut-off expect to make positive profits as they are able to cover fixed costs.

In order to ensure that the fdi cutoff productivity is higher than export productivity,i.e $\varphi^{fdi}_{ij} > \varphi^{ex}_{ij}$, we follow, [9] and assume that $f_{fdi}\mathcal{R}_{ij}^{\eta(\sigma-1)} > f_{ex}\tau^{\eta(\sigma-1)}$.

3 Aggregation and Equilibrium

In this section we present aggregation of cutoff productivity levels,Price Index exports and FDI sales, in country j. There is a continuum of prospective entrants that are the same ex-ante, to enter, they pay a sunk entry cost $f_{E_i} > 0$ units of labour,$w_j f_{ex}$ in the case of exporting firms or $w_j f_{fdi}$ in the case of FDI firms. Thereafter,firms independently draw productivity levels from a common distribution $g(\varphi) = k(\varphi_m in)^k \varphi^{-(k+1)}$ with positive support over $(0, \varphi_m ax)$ and a continuous cumulative distribution $G(\varphi)$,where $G(\varphi) = 1 - \left(\frac{\varphi_{min_i}}{\varphi}\right)^{\kappa}$.φ_{min} is the minimum productivity in the productivity distribution. Productivity is assumed to be Pareto distributed, the degree of firm heterogeneity is summarized bythe shape parameter $k > \sigma - 1$. This insures that the distribution of productivity draws finite variances. Since k is an inverse measure of variance, lower values of k correspond to greater firm heterogeneity(larger variance of firm productivity).

The Pareto distribution has a number of properties that make it analytically tractable hence its use on models of firm selection into export and FDI markets.From empirical evidence, it is a good approximation of the upper tail of distribution of firm sizes,this was first noted by [19]. Pareto distribution for US and European firms used to predict FDI was estimated by [9]. A key feature of a Pareto distributed random variable is that when truncated the random variable retains a Pareto distribution with the same shape parameter k. Therefore if entry is subject to an endogenous productivity cutoff, the distribution of the technologies that make the cut remains Pareto distributed. Another key feature of a Pareto distributed random variable is that power functions of this random variable are themselves Pareto distributed. Therefore, individual prices have a Pareto distribution, with a constant elasticity of demand, so do sales, hence firm size and variable profits are Pareto distributed.

In the same vein as [16] we let M denote the equilibrium mass of incumbent domestic firms in each country. Firms decisions to enter into domestic industry and or into foreign markets via exports and FDI are based on comparing expected profits and costs of entering the market. The probability of entry in the home market, exporting and into (conditional on successful entry) are given by the following equations respectively:

$$\theta_{iD} = 1 - G(\varphi_i^*) \tag{20}$$

$$\theta_{ex} = \frac{G(\varphi_{fdi}^*) - G(\varphi_{ex}^*}{1 - G(\varphi_i^*)} \tag{21}$$

$$\theta_{fdi} = \frac{1 - G(\varphi_{fdi}^*)}{1 - G(\varphi_i^*)} \tag{22}$$

Using these conditional probabilities of successful entry, the mass of firms that enter foreign country via exports and FDI are given by $M_{ex} = \theta_{ex} M$ and $M_{fdi} = \theta_{fdi} M$,respectively. Total mass of varieties available to consumers in each country is given by the total mass of firms competing in the country,

$$M = M + nM_{ex} + nM_{fdi} \tag{23}$$

Weighted productivity average

$$\widehat{\varphi} = \frac{1}{M}\left(M\widehat{\varphi}_{iD}^{\sigma-1} + nM_{ex}\tau^{1-\sigma}\widehat{\varphi}_{ex}^{\sigma-1} + nM_{fdi}\tau^{(1-\sigma)(1-\eta)}\widehat{\varphi}_{fdi}^{\sigma-1}\right)^{\frac{1}{\sigma-1}} \tag{24}$$

Aggregate price index as a function of weighted aggregate productivity and mass of varieties is given by $P = M^{\frac{1}{1-\sigma}} p(\widehat{\varphi})$.This price index is a decreasing as total mass of varieties increases and aggregate productivity increses. This implies that consumer welfare is given by $W = M^{\frac{1}{\sigma-1}}\rho\widehat{\varphi}$. From this welfare function, we have (a)variety gains;increase in consumer welfare with a rise in varieties, (b)productivity gains, higher weighted average produ ctivity implies that costs of production are lower hence lower prices are charged for consumer goods.

3.1 Free-Entry Condition

Prior to entry, potential entrants contemplate profits they would incurr if they enter and compare the expected profits to fixed costs of entry, f_{E_i}.In a stationary equilibrium as long as a firm is active it earns the same profis in each period. Ex ante, Free entry condition implies that expectation of future profits $\overline{\pi}$ conditional on successful entry must be equal to the fixed costs of entry f_{E_i}. Net value of entry $\overline{v}(\varphi)$ at time $(t = 0)$ given the probablity of dying in each period δ is

$$\overline{v}(\varphi) = E\left[\sum_{t=0}^{\infty}(1-\delta)^t\overline{\pi} - f_{E_i}\right] = \frac{1-G(\varphi^*)}{\delta}\overline{\pi} - f_{E_i} \tag{25}$$

According to the free entry condition, firms will enter until the net value of entry is driven to zero. Market shares of firms shrink as more firms enter the market until expected profits equal cost of entry.

Where the average revenues and profits across all domestic firms earned from both domestic, export and FDI revenues are given as:

$$\overline{r} = \pi_{iD}(\tilde{\varphi}) + n\theta_{ex}r_{ex}(\tilde{\varphi}) + n\theta_{fdi}r_{fdi}(\tilde{\varphi}) \tag{26}$$

Now taking into account the zero profit condition the above equation becomes

$$\overline{r} = \sigma(\overline{\pi} + f_{iD} + n\theta_{ex}f_{ex} + n\theta_{fdi}f_{fdi}) \tag{27}$$

$$\overline{\pi} = \pi_{iD}(\tilde{\varphi}) + n\theta_{ex}\pi_{ex}(\tilde{\varphi}) + n\theta_{fdi}\pi_{fdi}(\tilde{\varphi}) \tag{28}$$

These average Profits pin down the equilibrium mass of incumbent firms given as

$$M = \frac{R}{\overline{r}} = \frac{L}{\sigma(\overline{\pi} + f_{D_i} + n\theta_{ex}f_{ex} + n\theta_{fdi}f_{fdi}})\tag{29}$$

3.2 Labour Market clearing condition

The labour market clearing condition requires that total labour demand in domestic market equal labour total labour supply, i.e aggregate revenue(derived from consumers total expenditure on d ifferentiated goods) equal total payments to labour L. We then use the labour market clearing condition to solve for the mass of firms. The labour market clearing conding implies that total labour used for production and the entry cost must equal total labour endowment.We get the following expression for the mass of firms:

$$M_i = \frac{L_i(\sigma-1)(\varphi_{min_i}/\varphi_{ii})^k}{k\sigma f_{E_i}} L_i \tag{30}$$

3.3 Aggregation of Sales

Weconsider aggregation of export and FDI sales inorder to determine the internsive and extensive margins.

3.3.1 Export sales

The approach is to multiply the probability of export conditional on successsful entry by the export revenues.

$$X_{ij}^{ex} = \frac{G(\varphi_{fdi}^*) - G(\varphi_{ex}^*)}{1 - G(\varphi_i^*)} N_{ij}^{ex} \left(\int_{\varphi_{ij}^{ex}}^{\infty} \varphi^{\sigma-1} (\frac{\sigma}{\sigma-1} w_{ij}\tau_{ij})^{1-\sigma} \beta \frac{Y_j}{P_j^{1-\sigma}} dG(\varphi_{ij}^{ex}) \right) \tag{31}$$

Solving the integral and substituting for the export cutoff productivity yields the following decomposition for total exports into intensive and extensive margins (see appendix for derivations);

$$X_{ij}^{ex} = \underbrace{\left(\frac{\varphi_{min}}{\varphi_{ex}}\right)^k M_{ij}^{ex}}_{\text{extensive}} \underbrace{\left(\frac{\sigma k}{k-\sigma+1}\right) f_{ex} w_j}_{\text{intensive}} \tag{32}$$

At the ex tensive margins we have the number of firms that supply foreign market via exports and at the intensive margins we have the average sales of operating exporting firms. The effect of a change in trade costs on e xport sales also yields extensive and intensive margins as([5]for derivations);

$$\frac{dlnX_{ij}^{ex}}{dln\tau_{ij}} = -\underbrace{(\sigma-1)}_{\text{intensive}} - \underbrace{(k-\sigma+1)}_{\text{extensive}} \tag{33}$$

The same observations as [5] holds, Intensive margins are more sensitive to change in trade barriers asnd the extensive margins are less sensitive. Consider a situation where the elasticity of substitution is high, this implies tough competition between firms. Lowering trade barriers will induce new and less productive firms to enter, but low productivities implies that they are at a cost disadvantage translating into smaller market share. Consequently these low productive firms have a smaller impact on aggregate trade. However if elasti city of substitution is very low, competion is low and new entrants can capture largermarket resulting in larger impact on aggregate trade. Therefore, higher elasticity of substitution magnifies sensitivity of intensive margins to changes in trade costs, but sensitivity of extensive margins is less.

3.3.2 FDI or Multinational Sales

We proceed by multiplying the probability of entry into foreign market via FDI with revenues accrued from FDI, we get the following gravity equation

$$X_{ij}^{fdi} = \frac{1-G(\varphi_{fdi}^{*})}{1-G(\varphi_{i}^{*})} N_{ij}^{fdi} \left(\int_{\varphi_{ij}^{fdi}}^{\infty} \varphi^{\sigma-1} (\frac{\sigma}{\sigma-1})^{1-\sigma} [(\Re_{ij}\tau_{ij})^{\eta(\sigma-1)} - 1](w_{ij}\tau_{ij})^{1-\sigma} \beta \frac{Y_j}{P_j^{1-\sigma}} dG(\varphi_{ij}^{fdi}) \right) \tag{34}$$

Solving the integral and substituting for the FDI productivity cutoff, we get t he following decomosition of extensive and intesive margins of FDI (see appendix);

$$X_{ij}^{fdi} = \underbrace{\left(\frac{\varphi_{min}}{\varphi_{fdi}}\right)^{k} M_{ij}^{fdi}}_{\text{extensive}} \underbrace{\left(\frac{\sigma k}{k-\sigma+1}\right) w_j (f_{fdi} - f_{ex})}_{\text{intensive}} \tag{35}$$

In the same frame of mind as [11], the derivation of the overall effect of a change in variable trade barriers on total affilliate sales can be decomposed into intensive and extensive margins as;

$$\frac{dlnX_{ij}^{fdi}}{dln\tau_{ij}} = -\underbrace{(1-\eta)(\sigma-1)}_{\text{intensive}} - \underbrace{(k-\sigma+1)\chi_{fdi}}_{\text{extensive}} \tag{36}$$

Where χ_{fdi} is defined as the elasticity of FDI cutoff to variable trade barriers.

$$\chi_{fdi} = \frac{(\Re_{ij}\tau_{ij})^{\eta(\sigma-1)}(\eta-1)-1}{(\Re_{ij}\tau_{ij})^{\eta(\sigma-1)}-1} \tag{37}$$

In the case on FDI, the sensitivity of intensive margins to changes in trade barriers arises due to the intermediate inputs which are traded from country of origin of the foreign affiliate. The presence of this intrafirm trade implies that total FDI sales are inversely related to trade costs. At the extensive margins, higher trade costs leads to reduction in total FDI(multinational) sales. A higher elasticity of substitution in the presence of low trade costs induces new firms to enter foreign market via FDI provided they meet the cutoff productivity level of FDI. The higher elasticity o f sub stitution impliesfierce competition. Lowere productiities of those which meet the cutoff FDI productivity implies implies that they are at a cost disadvantage and only capture a small portion of the market, hence the impact of these low producivity firms on FDI sales is very small. However, a decrease in elasticity of substitution implies less competition and new FDI entrat firms can capture large market size with a large impact on FDI sales.

3.3.3 Aggregate Sales(from all operations, i.e both exports and FDI)

Aggregate sales from all operations are therefore a sum of the total ex port(trade) and FDI(multinational) sales, given by;

$$X_{ij}^{-k} = N_{ij}\frac{k}{k-\sigma+1}\left(\frac{Y_j}{P_j^{1-\sigma}}\right)^{1-\frac{k}{\sigma-1}}\left(\frac{\sigma}{\sigma-1}w_i\tau_{ij})\right)^{-k}(\sigma w_j)^{1-\frac{k}{\sigma-1}}\left(f_{ex}^{1-\frac{k}{\sigma-1}}+(\frac{f_{fdi}-f_{ex}}{(\Re\tau_{ij})^{\eta(\sigma-1)}-1})^{1-\frac{k}{\sigma-1}}\right) \tag{38}$$

3.4 Aggregation of Price Index

We define aggregate price, but decompose it into export price index and FDI price index (derivations in the appendix). This is to facilitate the derivation of the total welfare gains which we define in the proceeding section.

3.4.1 Trade only Price Index

$$P_j^{ex} = \frac{G(\varphi_{fdi}^*) - G(\varphi_{ex}^*)}{1 - G(\varphi_i^*)} N_{ij}^{ex} \left(\int_{\varphi_{ij}^{ex}}^{\infty} \varphi^{\sigma-1} (\frac{\sigma}{\sigma-1} w_{ij}\tau_{ij})^{1-\sigma} dG(\varphi_{ij}) \right)^{\frac{1}{1-\sigma}} \tag{39}$$

Evaluating the integral and substituting for cutoff productivities

$$P_j^{-k} = N_{ij} \frac{k}{k-\sigma+1} \left(\frac{1}{Y_j}\right)^{1-\frac{k}{\sigma-1}} \varphi_{min}^k \left(\frac{\sigma}{\sigma-1} w_i\tau_{ij}\right)^{-k} \left(\sigma w_j f_{ij}^{ex})\right)^{1-\frac{k}{\sigma-1}} \tag{40}$$

3.4.2 FDI only Price Index

$$P_{ij}^{fdi} = \frac{1 - G(\varphi_{fdi}^*)}{1 - G(\varphi_i^*)} N_{ij}^{fdi} \left(\int_{\varphi_{ij}^{fdi}}^{\infty} \varphi^{\sigma-1} (\frac{\sigma}{\sigma-1})^{1-\sigma} [(\Re_{ij}\tau_{ij})^{\eta(\sigma-1)} - 1](w_{ij}\tau_{ij})^{1-\sigma} dG(\varphi_{ij}) \right)^{\frac{1}{1-\sigma}} \tag{41}$$

Evaluating the integral and substituting for cutoff productivities

$$P_j^{-k} = N_{ij} \frac{k}{k-\sigma+1} \left(\frac{1}{Y_j}\right)^{1-\frac{k}{\sigma-1}} \varphi_{min}^k \left(\frac{\sigma}{\sigma-1} w_i\tau_{ij}\right)^{-k} \left(\sigma w_j (\frac{f_{fdi} - f_{ex}}{(\Re\tau_{ij})^{\eta(\sigma-1)} - 1})\right)^{1-\frac{k}{\sigma-1}} \tag{42}$$

. Aggregate price index from all operations are therefore a sum of the total export(trade) and FDI(multinational) price indices, given by;

$$\begin{aligned} P_{ij}^{1-\sigma} &= \frac{G(\varphi_{fdi}^*) - G(\varphi_{ex}^*)}{1 - G(\varphi_i^*)} N_{ij}^{ex} \left(\int_{\varphi_{ij}^{ex}}^{\infty} \varphi^{\sigma-1} (\frac{\sigma}{\sigma-1} w_{ij}\tau_{ij})^{1-\sigma} dG(\varphi_{ij}) \right) \\ &\quad + \frac{1 - G(\varphi_{fdi}^*)}{1 - G(\varphi_i^*)} N_{ij}^{fdi} \left(\int_{\varphi_{ij}^{fdi}}^{\infty} \varphi^{\sigma-1} (\frac{\sigma}{\sigma-1})^{1-\sigma} [(\Re_{ij}\tau_{ij})^{\eta(\sigma-1)} - 1](w_{ij}\tau_{ij})^{1-\sigma} dG(\varphi_{ij}) \right) \end{aligned}$$

Evaluating the Integral and substituting for the cut off productivities

$$P_j^{-k} = N_{ij} \frac{k}{k-\sigma+1} \left(\frac{1}{Y_j}\right)^{1-\frac{k}{\sigma-1}} \left(\frac{\sigma}{\sigma-1} (w_i\tau_{ij})\right)^{-k} (\sigma w_j)^{1-\frac{k}{\sigma-1}} \left(f(_{ij}^{ex})^{1-\frac{k}{\sigma-1}} + (\frac{f_{fdi} - f_{ex}}{(\Re\tau_{ij})^{\eta(\sigma-1)} - 1})^{1-\frac{k}{\sigma-1}} \right) \tag{43}$$

4 Welfare Analysis

We are interested in the aggregate effects of trade and FDI on the welfare measure. Welfare is given by the per capita value of real income accruing to consumers. Welfare depends on real labour income derived from export and foreign multinational affiliates in the domestic market and revenues generated by home country firms set up in foreign countries, this welfare measure is given given by:

$$W_j = \frac{Y_j}{L_j P_j} = \frac{w_j^{ex}}{P_j^{ex}} + \frac{w_j^{fdi}}{P_j^{fdi}} \tag{44}$$

Countries receiving inward FDI gain through an increase in GDP, initially through the FDI itself,and a positive multiplier effect on the economy so that the final increase in national income is greater than the initial injection of FDI. An initial change in aggregate demand can have a much greater final impact on equilibrium national income,this is known as the multiplier effect. It comes about because injections of new demand for goods and services into the circular flow of income can stimulate further rounds of spending in other words home country's spending on foreign varieties produced from foreign plants in the domestic market is the foreign country's income.

4.1 Expenditure Shares

Country j's welfare is expressed by [1] as a function of the share of expenditure that falls on domestically produced goods(which is equal to 1 minus the import penetration ratio. This share, λ_{jj} under autarky is 1,therefore total size of gains from openness will be equal to $1 - \lambda_j$.In the considered trade models (Armington [2] ,Eaton and Kortum [6], and Melitz [16]), the change in real income associated with a change in iceberg trade costs with only trade and without FDI is computed as:

$$\hat{W} = \hat{\lambda}_{jj}^{-\frac{1}{\epsilon}} \tag{45}$$

We proceed to derive change in real income associated with change in trade costs in an environment of both trade and FDI. First we determine the expenditure shares of total income of country j spent on foreign varieties, we give separate expositions of trade and FDI shares and finally the aggregate share on all foreign varies(both imports and FDI).

4.1.1 Trade Share(Income spent on Imports)

We present here an expression of total fraction of income of c ountry j spent on goods from country i (trade shares) as a function of wages and labour allocated in each country and parameters(see appendix for derivations)

$$\lambda_{ij}^{ex} = \frac{(L_i/f_{iE})\varphi_{min_i}^{k} w_i^{-\left(\frac{k\sigma-(\sigma-1)}{\sigma-1}\right)}(\tau_{ij})^{-k} f_{ij_{ex}}^{1-\frac{k}{\sigma-1}}}{\sum\limits_{v}(L_{vj}/f_{vE})\varphi_{min_v}^{k} w_v^{-\left(\frac{k\sigma-(\sigma-1)}{\sigma-1}\right)}(\tau_{vj})^{-k} f_{vj_{ex}}^{1-\frac{k}{\sigma-1}}} \tag{46}$$

4.1.2 FDI Expenditure Share

This is the total income spent on goods supplied by multinationals in the host country (see appendix for derivations)

$$\lambda_{ij}^{fdi} = \frac{(L_i/f_{iE})\varphi_{min_i}^{k} w_i^{-\left(\frac{k\sigma-(\sigma-1)}{\sigma-1}\right)}(\tau_{ij})^{-k} f_{ij_{ex}}^{1-\frac{k}{\sigma-1}}(\mathcal{R}_{vj}\tau_{ij})^{\mu(\sigma-1)}-1)^{1-\frac{k}{\sigma-1}}}{\sum\limits_{v}(L_{vj}/f_{vE})\varphi_{min_v}^{k} w_v^{-\left(\frac{k\sigma-(\sigma-1)}{\sigma-1}\right)}(\tau_{vj})^{-k} f_{vj_{ex}}^{1-\frac{k}{\sigma-1}}(\mathcal{R}_{vj}\tau_{vj})^{\mu(\sigma-1)}-1)^{1-\frac{k}{\sigma-1}}} \tag{47}$$

In both the trade and FDI share, the exponent on the wages differs from the exponent on trade costs due to the assumption that fixed exporting costs are incurred in terms of labour in the source country.

4.1.3 Aggregate Expenditure share

The aggregate expenditure share on all foreign varieties is given by adding the last two expressions for trade and FDI expenditure shares.

4.2 Welfare Derivation

We use the expenditure shares derived in equations (46);trade expenditure share and (47);FDI expenditure share to write the generalised welfare derived in equation (44) as a function of expenditure shares and elasticities. Under Pareto assumption knowing a country's domestic share of trade and FDI and shape parameter of the productivity distribution k is sufficient to determine welfare gains from openness. To achieve this, we first write the export price index in equation (40) and FDI price index in equation (42) as a function of country j's share of trade(FDI) with itself, its export(FDI) wage, the export(FDI) labour allocation and parameters.

4.2.1 Welfare derived from Trade

$$\left(P_j^{ex}\right)^{-k} = \sum_i (L_i/f_E)\varphi_{min_i}^k w_i^{-k}(\tau_{ij})^{-k} f_{ex}^{1-\frac{k}{\sigma-1}} \left(\frac{w_j^{ex}}{Y_j^{ex}}\right)^{1-\frac{k}{\sigma-1}} \left(\frac{\sigma}{\sigma-1}\right)^{-k} \sigma^{-\frac{k}{\sigma-1}} \frac{\sigma-1}{k-\sigma+1} \tag{48}$$

We use the expression of country j's expenditure with itself, λ_{jj}^{ex} and the expression for income derived from export sales, $Y_j^{ex} = w_j^{ex} L_j^{ex}$

$$W_j^{ex} = \left(\frac{w_j^{ex}}{P_j^{ex}}\right) = \left[\left(\frac{\sigma}{\sigma-1}\right)^{-k} \sigma^{-\frac{k}{\sigma-1}} \frac{\sigma-1}{k-\sigma+1} \varphi_{min_j}^k f_{ex}^{1-\frac{k}{\sigma-1}} f_E\right]^{\frac{1}{k}} \left(L_j^{ex}\right)^{\frac{1}{\sigma-1}} \left(\lambda_{jj}^{ex}\right)^{-\frac{1}{k}} \tag{49}$$

We summarize the welfare gains from trade, measured as the welfare ratio between the open and closed economies(the domestic trade shares in the closed economy are fixed at $\lambda^{closed} = 1$):

$$\widehat{W}_j^{ex} = \frac{W_j^{open}}{W_j^{closed}} = \widehat{\lambda}_{j^{ex}}^{-\frac{1}{k}} \tag{50}$$

The change in real income derived from trade associated with a change in the country's expenditures on domestic goods.

4.2.2 Welfare Derived from FDI

$$\begin{aligned} \left(P_j^{fdi}\right)^{-k} &= \sum_i (L_i/f_{iE})\varphi_{min_i}^k w_i^{-k}(\tau_{ij})^{-k}[(\mathcal{R}_{ij}\tau_{ij})^{\mu(\sigma-1)} - 1]^{-\left(1-\frac{k}{\sigma-1}\right)} \\ & \quad (f_{fdi} - f_{ex})^{1-\frac{k}{\sigma-1}} \left(\frac{w_j^{fdi}}{Y_j^{fdi}}\right)^{1-\frac{k}{\sigma-1}} \left(\frac{\sigma}{\sigma-1}\right)^{-k} \sigma^{-\frac{k}{\sigma-1}} \\ & \quad \frac{\sigma-1}{k-\sigma+1} \end{aligned}$$

Now we use the expression of country j's expenditure with itself, λ_{jj}^{fdi} and the expression for income derived from FDI, $Y_j^{fdi} = w_j^{fdi} L_j^{fdi}$

$$W_j^{fdi} = \left(\frac{w_j^{fdi}}{P_j^{fdi}}\right) = \left[\left(\frac{\sigma}{\sigma-1}\right)^{-k} \sigma^{-\frac{k}{\sigma-1}} \frac{\sigma-1}{k-\sigma+1} \varphi_{min_j}^{k} (f_{fdi} - f_{ex})^{1-\frac{k}{\sigma-1}} f_{jE}^{-1}\right]^{\frac{1}{k}} (L_j^{fdi})^{\frac{1}{\sigma-1}} [\mathcal{R}_j^{\mu(\sigma-1)} - 1]^{\frac{(\sigma-1)-k}{(\sigma-1)k}} \left(\lambda_{jj}^{fdi}\right)^{-\frac{1}{k}}$$

Notice that an increase in marginal cost savings $[\mathcal{R}_j^{\eta(\sigma-1)} - 1]$, increases welfare with elasticity $\frac{1}{\sigma-1} - \frac{1}{k}$. A dec line in fixed costs of entryf_{E_i} increases the welfare gains with elasticity $\frac{1}{k}$, this is because lower fixed costs of entry leads to more firm entry, while number of operating firms remains constant. This implies more firm selection, since k is an inverse mearsure of productivity dispersion, the benefits of this firm selection are increasing with lower values of k,(i.e.,more firm heteregeneity). In a similar manner a change in welfare can be expressed as a function of domestic expenditure on FDI as

$$\widehat{W}_j^{fdi} = \frac{W_j^{open}}{W_j^{closed}} = [\hat{\mathcal{R}}_{jj}^{\mu(\sigma-1)} - 1]^{\frac{(\sigma-1)-k}{(\sigma-1)k}} \widehat{\lambda}_{jfdi}^{-\frac{1}{k}} \tag{51}$$

Total Welfare(Gains from Openness) Gains from openness (gains from trade and FDI) as specified equation (45) as depend on total real income derived from trade and FDI.Therefore the change in real income associated with a change in trade costs (for both trade and FDI) is given by

$$\begin{aligned} \widehat{W}_j &= \widehat{W}_j^{ex} + \widehat{W}_j^{fdi} \\ &= \widehat{\lambda}_{jex}^{-\frac{1}{k}} + [\hat{\mathcal{R}}_j^{\mu(\sigma-1)} - 1]^{\frac{(\sigma-1)-k}{(\sigma-1)k}} \widehat{\lambda}_{jfdi}^{-\frac{1}{k}} \end{aligned}$$

From this equation we see that, change in welfare depends on welfare derived from FDI and trade. We see that FDI provides an additional source of change in real income.. From this equation we deduce that decreasing marginal cost savings will decrease welfare gains from FDI. Where marginal cost savings are equal to zero firms make zero profits from FDI, the welfare derived from FDI becomes zero resulting only in trade gains $\widehat{W}_j^{ex}$ derived by ACR [1].

Country j's gains from openness C_j is the absolute value of the percentage change in real income associated with moving to a counterfactual equilibrium, this is represented as;

$$C_j = 1 - \left[\widehat{\lambda}_{jex}^{-\frac{1}{k}} + [\hat{\mathcal{R}}_j^{\mu(\sigma-1)} - 1]^{\frac{(\sigma-1)-k}{(\sigma-1)k}} \widehat{\lambda}_{jfdi}^{-\frac{1}{k}}\right] \tag{52}$$

5 Conclusion

In this paper, we have revisited the welfare gains from trade for new trade models in the presence of FDI. Provided there are positive marginal cost savings and positive profits from FDI, we get additional gains from FDI. Therefore omission of FDI revenues underestimates gains from openness relative to autarky. We anticipate that the bias is largest in economies characterized by high FDI revenue, mostly developed countries. The multinational affiliates used intermediate iputs provided via intra firm trade,this gives us insides about the interaction of trade and mutinationals. Whenever $\mu = 1$,there is no intrafirm trade of intermediate goods hence trade and FDI are substitutes as in the case of [9])in which FDI was favoured whenever trade costs are high(tarriff-jumping). However when $\mu = 0$,an increase in trade barriers leads to costly intrafirm firm trade of affiliates hence, affilliate profits deminish. Therefore for low values of μ, accessing foreign markets via FDI is most preffered, for high values of μ firm will access foreign market via exports.For intermediate values of μ trade facilitates multinationl production via intermediate inputs and FDI(multinational production) boots trade.

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6 Appendix

6.1 Aggregation Price Index

6.1.1 Trade Only Price Index

$$P_j^{ex} = \frac{G(\varphi_{fdi}^*) - G(\varphi_{ex}^*)}{1 - G(\varphi_i^*)} N_{ij}^{ex} \left(\int_{\varphi_{ij}^{ex}}^{\infty} \varphi^{\sigma-1} (\frac{\sigma}{\sigma - 1} w_{ij} \tau_{ij})^{1-\sigma} dG(\varphi_{ij}) \right)^{\frac{1}{1-\sigma}} \tag{53}$$

Under assumption of Common Pareto distribution this becomes

$$P_j^{ex} = N_{ij}^{ex} \left(\int_{\varphi_{ij}^{fdi}}^{\varphi_{ij}^{ex}} \varphi^{\sigma-1} (\frac{\sigma}{\sigma - 1} w_{ij} \tau_{ij})^{1-\sigma} k \varphi_{min}^k \varphi^{k+1} d(\varphi_{ij}))^{1-\sigma} \right)^{\frac{1}{1-\sigma}} \tag{54}$$

Evaluating the integral and substituting for cutoff productivities

$$P_j^{-k} = N_{ij} \frac{k}{k - \sigma + 1} \left(\frac{1}{Y_j} \right)^{1 - \frac{k}{\sigma - 1}} \varphi_{min}^k \left(\frac{\sigma}{\sigma - 1} w_i \tau_{ij} \right)^{-k} \left(\sigma w_j f_{ij}^{ex}) \right)^{1 - \frac{k}{\sigma - 1}} \tag{55}$$

.

6.1.2 FDI only Price Index

$$P_{ij}^{fdi} = \frac{1 - G(\varphi_{fdi}^*)}{1 - G(\varphi_i^*)} N_{ij}^{fdi} \left(\int_{\varphi_{ij}^{fdi}}^{\infty} \varphi^{\sigma-1} (\frac{\sigma}{\sigma - 1})^{1-\sigma} [(\Re_{ij} \tau_{ij})^{\eta(\sigma-1)} - 1] (w_{ij} \tau_{ij})^{1-\sigma} dG(\varphi_{ij}) \right)^{\frac{1}{1-\sigma}} \tag{56}$$

Under assumption of Common Pareto distribution this becomes

$$P_{ij}^{fdi} = N_{ij}^{fdi} \left(\int_{\varphi_{ij}^{fdi}}^{\infty} \varphi^{\sigma-1} (\frac{\sigma}{\sigma - 1} w_{ij} \tau_{ij})^{1-\sigma} [(\Re_{ij} \tau_{ij})^{\eta(\sigma-1)} - 1] k \varphi_{min}^k \varphi^{k+1} d(\varphi_{ij}))^{1-\sigma} \right)^{\frac{1}{1-\sigma}} \tag{57}$$

Evaluating the integral and substituting for cutoff productivities

$$P_j^{-k} = N_{ij}\frac{k}{k-\sigma+1}\left(\frac{1}{Y_j}\right)^{1-\frac{k}{\sigma-1}}\varphi_{min}^{k}\left(\frac{\sigma}{\sigma-1}w_i\tau_{ij}\right)^{-k}\left(\sigma w_j(\frac{f_{fdi}-f_{ex}}{(\Re\tau_{ij})^{\eta(\sigma-1)}-1})\right)^{1-\frac{k}{\sigma-1}} \tag{58}$$

.

6.1.3 Aggregate price index of all foreign varieties

For the Aggregate price index of all foreign varieties supplied via exports and FDI we start with the following integral

$$\begin{aligned}P_{ij}^{1-\sigma} \quad = \quad & \frac{G(\varphi_{fdi}^{*})-G(\varphi_{ex}^{*})}{1-G(\varphi_{i}^{*})}N_{ij}^{ex}\left(\int_{\varphi_{ij}^{ex}}^{\infty}\varphi^{\sigma-1}(\frac{\sigma}{\sigma-1}w_{ij}\tau_{ij})^{1-\sigma}dG(\varphi_{ij})\right)\\ & +\frac{1-G(\varphi_{fdi}^{*})}{1-G(\varphi_{i}^{*})}N_{ij}^{fdi}\left(\int_{\varphi_{ij}^{fdi}}^{\infty}\varphi^{\sigma-1}(\frac{\sigma}{\sigma-1})^{1-\sigma}[(\Re_{ij}\tau_{ij})^{\eta(\sigma-1)}-1](w_{ij}\tau_{ij})^{1-\sigma}dG(\varphi_{ij})\right)\end{aligned}$$

$$\begin{aligned}P_{ij}^{1-\sigma} \quad = \quad & N_{ij}^{ex}\left(\int_{\varphi_{ij}^{ex}}^{\infty}\varphi^{\sigma-1}(\frac{\sigma}{\sigma-1}w_{ij}\tau_{ij})^{1-\sigma}k\varphi_{min}^{k}\varphi^{k+1}d(\varphi_{ij})\right)\\ & +N_{ij}^{fdi}\left(\int_{\varphi_{ij}^{fdi}}^{\infty}\varphi^{\sigma-1}(\frac{\sigma}{\sigma-1})^{1-\sigma}[(\Re_{ij}\tau_{ij})^{\eta(\sigma-1)}-1](w_{ij}\tau_{ij})^{1-\sigma}k\varphi_{min}^{k}\varphi^{k+1}d(\varphi_{ij})\right)\end{aligned}$$

Evaluating the Integral and substituting for the cut off productivities

$$P_j^{-k} = N_{ij}\frac{k}{k-\sigma+1}\left(\frac{1}{Y_j}\right)^{1-\frac{k}{\sigma-1}}\left(\frac{\sigma}{\sigma-1}(w_i\tau_{ij})\right)^{-k}(\sigma w_j)^{1-\frac{k}{\sigma-1}}\left(f(_{ij}^{ex})^{1-\frac{k}{\sigma-1}}+(\frac{f_{fdi}-f_{ex}}{(\Re\tau_{ij})^{\eta(\sigma-1)}-1})^{1-\frac{k}{\sigma-1}}\right) \tag{59}$$

6.2 Aggregation of Sales

6.2.1 Export sales

$$X_{ij}^{ex} = \frac{G(\varphi_{fdi}^*) - G(\varphi_{ex}^*)}{1 - G(\varphi_i^*)} N_{ij}^{ex} \left(\int_{\varphi_{ij}^{ex}}^{\infty} \varphi^{\sigma-1} (\frac{\sigma}{\sigma-1} w_{ij}\tau_{ij})^{1-\sigma} \beta \frac{Y_j}{P_j^{1-\sigma}} dG(\varphi_{ij}^{ex}) \right) \tag{60}$$

Under the assumption of a common Pareto distribution for all countries, this becomes:

$$X_{ij}^{ex} = N_{ij}^{ex} \left(\int_{\varphi_{ij}^{ex}}^{\infty} \varphi^{\sigma-1} (\frac{\sigma}{\sigma-1} w_{ij}\tau_{ij})^{1-\sigma} \beta \frac{Y_j}{P_j^{1-\sigma}} k \varphi_{min}^k \varphi^{k+1} d(\varphi_{ij}^{ex}) \right) \tag{61}$$

Evaluating the integrals we get

$$X_{ij}^{ex} = \underbrace{\left(\frac{k}{k-\sigma+1} \varphi_{min}^{\sigma-1} \right) M_{ij}^{ex}}_{\text{supply capacity}} \underbrace{\left(\frac{\varphi_{min}}{\varphi_{ex}} \right)^{-k-\sigma+1} \frac{Y_j}{P_j^{1-\sigma}} (\frac{\sigma}{1-\sigma} w_{ij}\tau_{ij})^{1-\sigma}}_{\text{market capacity}} \tag{62}$$

Using the export productivity cut-off the last equation above can be re-written as:

$$X_{ij}^{ex} = M_{ij}^{ex} \frac{k}{k-\sigma+1} \left(\frac{Y_j}{P_j^{1-\sigma}} \right)^{\frac{k}{\sigma-1}} \left(\frac{\sigma}{\sigma-1} w_i \tau_{ij}) \right)^{-k} \left(\sigma w_j f_{ij}^{ex}) \right)^{1-\frac{k}{\sigma-1}} \tag{63}$$

Equivalently this can be re-written as:

$$X_{ij}^{ex} = \underbrace{\left(\frac{\varphi_{min}}{\varphi_{ex}} \right)^k M_{ij}^{ex}}_{\text{extensive}} \underbrace{\left(\frac{\sigma k}{k-\sigma+1} \right) f_{ex} w_j}_{\text{intensive}} \tag{64}$$

The effect of trade costs on export sales also yields the extensive and intensive margins as

$$\frac{dlnX_{ij}^{ex}}{dln\tau_{ij}} = -\underbrace{(\sigma-1)}_{\text{intensive}} - \underbrace{(k-\sigma+1)}_{\text{extensive}} \tag{65}$$

6.2.2 FDI or Multinational Sales

$$X_{ij}^{fdi} = \frac{1-G(\varphi_{fdi}^*)}{1-G(\varphi_i^*)} N_{ij}^{fdi} \left(\int_{\varphi_{ij}^{fdi}}^{\infty} \varphi^{\sigma-1} (\frac{\sigma}{\sigma-1})^{1-\sigma} [(\Re_{ij}\tau_{ij})^{\eta(\sigma-1)} - 1](w_{ij}\tau_{ij})^{1-\sigma} \beta \frac{Y_j}{P_j^{1-\sigma}} dG(\varphi_{ij}^{fdi}) \right) \tag{66}$$

Under the assumption of a common Pareto distribution for all countries, this becomes:

$$X_{ij}^{fdi} = N_{ij}^{fdi} \left(\int_{\varphi_{ij}^{fdi}}^{\infty} \varphi^{\sigma-1} (\frac{\sigma}{\sigma-1})^{1-\sigma} [(\Re_{ij}\tau_{ij})^{\eta(\sigma-1)} - 1](w_{ij}\tau_{ij})^{1-\sigma} \beta \frac{Y_j}{P_j^{1-\sigma}} k\varphi_{min}^k \varphi^{k+1} d(\varphi_{ij}^{fdi}) \right) \tag{67}$$

Evaluating the integrals we get

$$X_{ij}^{fdi} = \underbrace{\left(\frac{k}{k-\sigma+1} \varphi_{min}^{\sigma-1} \right) M_{ij}^{fdi}}_{\text{supply capacity}} \underbrace{\left(\frac{\varphi_{min}}{\varphi_{fdi}} \right)^{-k-\sigma+1} \frac{Y_j}{P_j^{1-\sigma}} [(\Re_{ij}\tau_{ij})^{\eta(\sigma-1)} - 1](\frac{\sigma}{1-\sigma} w_{ij}\tau_{ij})^{1-\sigma}}_{\text{market capacity}} \tag{68}$$

Using the FDI productivity cut-off the last equation above can be re-written as:

$$X_{ij}^{fdi} = N_{ij}^{fdi} \frac{k}{k-\sigma+1} \left(\frac{Y_j}{P_j^{1-\sigma}} \right)^{1-\frac{k}{\sigma-1}} \left(\frac{\sigma}{\sigma-1} w_i \tau_{ij}) \right)^{-k} \left(\sigma w_j (\frac{f_{fdi} - f_{ex}}{(\Re\tau_{ij})^{\eta(\sigma-1)} - 1}) \right)^{1-\frac{k}{\sigma-1}} \tag{69}$$

In the same spirit as the trade case(export) this can be re-written as;

$$X_{ij}^{fdi} = \underbrace{\left(\frac{\varphi_{min}}{\varphi_{fdi}} \right)^k M_{ij}^{fdi}}_{\text{extensive}} \underbrace{\left(\frac{\sigma k}{k-\sigma+1} \right) w_j (f_{fdi} - f_{ex})}_{\text{intensive}} \tag{70}$$

Following Irarrabal(2012)The overall effect of an increase in variable trade barriers on total affiliate sales can also be decomposed into intensive and extensive margin as ;

$$\frac{dlnX_{ij}^{fdi}}{dln\tau_{ij}} = -\underbrace{(1-\eta)(\sigma-1)}_{\text{intensive}} - \underbrace{(k-\sigma+1)\chi_{fdi}}_{\text{extensive}} \tag{71}$$

Where χ_{fdi} is defines as the elasticity of FDI cutoff to variable trade barriers.

$$\chi_{fdi} = \frac{(\Re_{ij}\tau_{ij})^{\eta(\sigma-1)}(\eta-1)-1}{(\Re_{ij}\tau_{ij})^{\eta(\sigma-1)}-1} \tag{72}$$

6.2.3 Aggregate Sales(from all operations, i.e both exports and FDI)

$$X_{ij}^{-k} = N_{ij}\frac{k}{k-\sigma+1}\left(\frac{Y_j}{P_j^{1-\sigma}}\right)^{1-\frac{k}{\sigma-1}}\left(\frac{\sigma}{\sigma-1}w_i\tau_{ij})\right)^{-k}(\sigma w_j)^{1-\frac{k}{\sigma-1}}\left(f_{ex}^{1-\frac{k}{\sigma-1}}+(\frac{f_{fdi}-f_{ex}}{(\Re\tau_{ij})^{\eta(\sigma-1)}-1})^{1-\frac{k}{\sigma-1}}\right) \tag{73}$$

6.3 Expenditure shares

6.3.1 Trade Share(Income spent on Imports

$$\lambda_{ij}^{ex} = \frac{X_{ij}^{ex}}{\sum\limits_v X_{vj}} = \frac{\left(\frac{\varphi_{ii}^*}{\varphi_{ij}^*}\right)^k N_i w_i f_{ij_{ex}}\frac{\sigma k}{k-\sigma+1}}{\sum\limits_v\left(\frac{\varphi_{ii}^*}{\varphi_{vj}^*}\right)^k N_i w_i f_{vj_{ex}}\frac{\sigma k}{k-\sigma+1}} \tag{74}$$

Using the solution for the equilibrium mass of firms derived from the labour market clearing condition and free entry condition we get

$$\lambda_{ij}^{ex} = \frac{(L_i/f_{iE})\varphi_{min_i}^k\left(\varphi_{ij}^*\right)^{-k}w_i f_{ij_{ex}}}{\sum\limits_v(L_{vj}/f_{vE})\varphi_{min_v}^k\left(\varphi_{vj}^*\right)^{-k}w_v f_{vj_{ex}}} \tag{75}$$

Using the export cutoff productivity

$$\lambda_{ij}^{ex} = \frac{(L_i/f_{iE})\varphi_{min_i}^k w_i^{-\left(\frac{k\sigma-(\sigma-1)}{\sigma-1}\right)}(\tau_{ij})^{-k}f_{ij_{ex}}^{1-\frac{k}{\sigma-1}}}{\sum\limits_v(L_{vj}/f_{vE})\varphi_{min_v}^k w_v^{-\left(\frac{k\sigma-(\sigma-1)}{\sigma-1}\right)}(\tau_{vj})^{-k}f_{vj_{ex}}^{1-\frac{k}{\sigma-1}}} \tag{76}$$

6.3.2 FDI Expenditure Share

$$\lambda_{ij}^{fdi} = \frac{X_{ij}^{fdi}}{\sum\limits_v X_{vj}} = \frac{\left(\frac{\varphi_{ii}^*}{\varphi_{ij}^*}\right)^k N_i w_i(f_{ij_{fdi}}-f_{ij_{ex}})\frac{\sigma k}{k-\sigma+1}}{\sum\limits_v\left(\frac{\varphi_{ii}^*}{\varphi_{vj}^*}\right)^k N_i w_i(f_{vj_{fdi}}-f_{vj_{ex}})\frac{\sigma k}{k-\sigma+1}} \tag{77}$$

Using the solution for the equilibrium mass of firms derived from the labour market clearing condition and free entry condition we get

$$\lambda_{ij}^{fdi} = \frac{(L_i/f_{iE})\varphi_{min_i}^k \left(\varphi_{ij}^*\right)^{-k} w_i(f_{ij_{fdi}} - f_{ij_{ex}})}{\sum_v (L_{vj}/f_{vE})\varphi_{min_v}^k \left(\varphi_{vj}^*\right)^{-k} w_v(f_{vj_{fdi}} - f_{vj_{ex}})} \tag{78}$$

Using the export FDI productivity

$$\lambda_{ij}^{fdi} = \frac{(L_i/f_{iE})\varphi_{min_i}^k w_i^{-\left(\frac{k\sigma-(\sigma-1)}{\sigma-1}\right)}(\tau_{ij})^{-k} f_{ij_{ex}}^{1-\frac{k}{\sigma-1}} (\Re_{vj}\tau_{ij})^{\eta(\sigma-1)} - 1)^{1-\frac{k}{\sigma-1}}}{\sum_v (L_{vj}/f_{vE})\varphi_{min_v}^k w_v^{-\left(\frac{k\sigma-(\sigma-1)}{\sigma-1}\right)}(\tau_{vj})^{-k} f_{vj_{ex}}^{1-\frac{k}{\sigma-1}} (\Re_{vj}\tau_{vj})^{\eta(\sigma-1)} - 1)^{1-\frac{k}{\sigma-1}}} \tag{79}$$

6.3.3 Aggregate Expenditure share

The aggregate expenditure share on all foreign varieties is given by adding the last two expressions for trade and FDI expenditure shares