Trade, Education, and The Shrinking Middle Class*

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October 18, 2011

Abstract

We develop a new model of trade in which educational institutions drive comparative advantage and determine the distribution of human capital within and across countries. Our framework exploits a multiplicity of sectors and continuous support of possible human capital choices to demonstrate that freer trade can induce crowding out of the middle occupations towards the skill acquisition extremes in one country and simultaneous expansion of middle-income industries in another. Individual gains from trade may be non-monotonic in workers' ability, and middle ability agents can lose the most from trade liberalization. Endogenizing trade and education policy, we find that targeted education subsidies are more effective than tariffs as a means to preserve "middle class" jobs, while uniform educational subsidies are of little consequence.

Keywords: Trade and Education Policy, Skill Acquisition, Education, Income Distribution

JEL Classifications: F11, F13, F15, F16

*We are grateful to John McLaren, Andrés Rodríguez-Clare, Sarah Turner, and seminar participants at Kiel, Trinity University, Copenhagen Business School, and the University of Virginia, St. Louis Federal Reserve, 2008 NBER ITI Summer Institute, 2009 AEA meetings, 2009 Midwest International Economics Group, and 2009 Belgian Trade Workshop for thoughtful suggestions. Maarten Goos and Alan Manning generously shared their figures, which help motivate the model.

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1 Introduction

Politicians tend to portray education as a universal panacea for rising income inequality and perceived competition with foreign exporters – a silver bullet with which the industrialized world will be able to maintain a thriving middle class and ever-greater standards of living. At the same time, popular sentiment reflects growing unease with individuals' economic prospects and a perception that even a solid education no longer guarantees a good job, secure career path, or membership in the middle class. In this paper, we explore the source of the disconnect between political rhetoric and and the apparent reality, developing a novel model of how trade and educational institutions interact to determine individuals' skill acquisition decisions, the distribution of income, and ultimately the welfare consequences of trade liberalization.

It comes as no surprise to anyone who reads the daily papers that the data square more with the gloomy public view than with heady political aspirations. A series of recent studies find evidence of both job and wage polarization in developed countries. In their seminal contribution, Goos and Manning (2007) demonstrate a hollowing out of middle class employment in the United Kingdom between 1979 and 1999. Figure 1, reproduced with permission below, shows job growth at both the bottom and top two 'job quality' deciles, with a simultaneous reduction in employment for the occupations in between. Autor, Levy, and Murnane (2003) and Autor, Katz, and Kearney (2006) document similar findings for the United States, while Falvay, Greenaway, and Silva (2008) find the same trend in Portugal.[1]

The evidence points clearly to a demand-side story – at least in the rich world–

[1]Broda and Romalis (2009) provide further evidence of a middle-class real wage decline relative to low and high income groups through relative prices of consumption bundles under non-homothetic preferences: prices for goods bought by low-income families have fallen *more* than prices for consumption bundles for middle-income families; high-income groups have seen the slowest price declines, but their nominal wages have also risen the fastest.

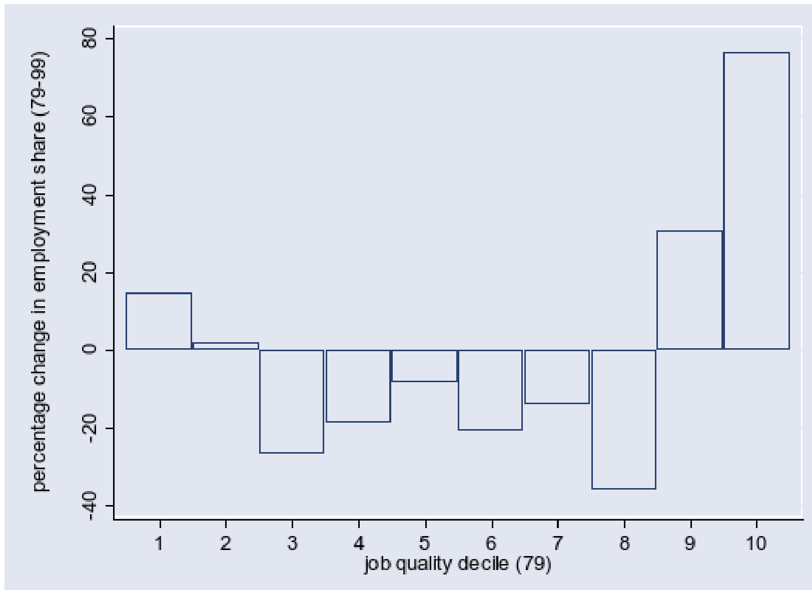

Figure 1: Non-Monotonic Changes in Employment (Goos and Manning (2007))

given that the same shrinking middle class phenomenon is revealed in both wage levels (price) as well as employment (quantity). Yet we submit that the demand side is unlikely a story of technological change alone, as has been suggested by Autor, Katz, and Kearney (2006), but is (also) crucially tied to countries' comparative advantage, relative educational structures, and globalization writ large (trade, offshoring, outsourcing). Simply look to the developing world for evidence. While income inequality is growing in many poor countries – particularly where natural resources are controlled by a small political elite and commodity prices have been rising – the broader trend seems to push in the opposite direction. In the two most comprehensive studies to date, Ravallion (2009) and a recent special report in The Economist magazine[2], document a dramatic expansion of the middle classes in the developing world in recent decades. Were technology the sole force at play, income inequality and the vanishing middle class phenomenon should be evident in the developing world, too. [3]

[2] "Burgeoning Bourgeoisie: A special Report on the New Middle Classes in Emerging Markets," February 14, 2009

[3] Of course, we are not suggesting that technology plays no role – rather that it is not the *only*

We posit that differences in workers' productivities in the developing world – particularly via their skill sets and educational attainment – may play a crucial role in shaping demand for skilled workers in the industrialized world. At the same time, workers' skill acquisition is endogenous in both the rich and poor worlds – and depend on local institutions, wages, and (thus) trade openness. The upshot: we need sophisticated understanding of how educational institutions and trade interact in order to understand just *how* and to what extent education can serve as the 'magic elixir' politicians seem to think it is.

To this end, we develop a new model in which trade and educational institutions interact to determine individuals' skill acquisition decisions, the distribution of income, and ultimately the welfare consequences of trade liberalization. In so doing, this paper makes three main contributions: first, our continuum of sectors modeling structure allows for non-montonic skill change in response to trade liberalization, which can rationalize the evidence outlined above. Second, our model highlights international differences in educational institutions as an additional source of comparative advantage. Finally, as educational institutions are largely a function of government policy, our model offers a novel lens through which to compare educational and trade policy.

Our framework features a continuum of heterogeneous agents who choose among a continuum of occupational sectors (or tasks), each of which requires a unique set of skills for employment. Each occupation is in turn used in the production of a particular intermediate product or service. Workers' wages are determined by sectoral technology (which we keep simple) and intermediate good/task prices — and thus indirectly by trade openness — while the cost of human capital acquisition is determined by individual characteristics and the country-specific structure of educational institutions and policies. Faced with the resulting incentive structure, agents of dif-

explanation.

ferent inherent ability levels self-select into sectors by investing in the corresponding human capital.

Comparative advantage in our framework is driven by international differences in educational institutions, and the resulting differential costs of skill acquisition. Trade liberalization — by affecting wages and hence incentives — leads to a remapping of agents to sectors, as would changes in trading partners' technologies, trade costs, or educational institutions that influence the cost of skill acquisition. The resulting shift in the demographics of skill composition can take many forms. One plausible and particularly salient scenario in line with the diagram below is the crowding out of the middle class towards the skill acquisition extremes, which can be brought about by increasing foreign competition in medium-level intermediate goods or tasks. In response to increased foreign competition in mid-range intermediate sectors, some moderate ability agents will invest in higher types of human capital, while others will optimally invest in lesser skills. More generally, we find that only in special cases would all agents' human capital decisions respond monotonically to such changes. While the aggregate gains from trade are shown to be positive, the distributional consequences are complex and generally non-monotonic.

Turning to the area of policy analysis, the model sheds light on the potential differential impact of strengthening educational institutions versus the effects of trade protection. In general, government subsidies to education or similar institutional improvements that decrease the cost of skill acquisition will influence workers' human capital decisions and thus the pattern of comparative advantage and trade. Targeted trade protection can have a similar effect by buoying the wages in import-sensitive sectors relative to the rest of the economy, but the concomitant consumption distortions imposed by tariffs make education policy a more efficient policy tool for redistributing income and human capital. We find additionally that if the government has an explicit preference for a 'strong middle class' then uniform educational investment will serve little purpose. Instead, educational incentives would need to induce skill upgrading among the least able workers through more pronounced subsidization and reform at the most basic levels, while allowing for skill downgrading (or at least slower upgrading) among the highly educated elite. Such a policy would, however, channel workers into the import-competing 'middle class' sectors, and thus may prove untenable over the longer term.

Our approach in this paper is motivated in part by recent work on the labor market that analyzes the diverging development of different segments of the labor market. As noted earlier, Autor, Levy, and Murnane (2003), Goos and Manning (2007), and Falvay, Greenaway, and Silva (2008) document that employment growth has been non-monotonic across sectors in that it is positive at the low and high ends of the labor market, but negative in the middle. While Autor and Dorn (2007) propose a simple three-sector model to explain this divergence, we provide a theoretical framework that is considerably more general and links the shift in employment and human capital to trade liberalization.

A second motivation of our work is the two-sector limitation of most work in the trade literature that endogenizes human capital formation. In earlier models, including Blanchard and Willmann (2011), workers can be either skilled or unskilled. The opening up of trade will — under such a binary setup — induce either skill upgrading (in a country with comparative advantage in skill intensive products) or skill downgrading (in countries with comparative advantage in 'basic' goods) through customary Stolper-Samuelson forces, but not both. Although analytically parsimonious, such a binary approach oversimplifies the process of skill acquisition. Not only are the educational policy implications implausibly stark, the economic prediction denies an important empirical regularity: that *within* countries freer trade causes some workers to 'sort down' — often into low-skill menial jobs — while others simultaneously 'sort up' into higher skill jobs. The modeling strategy we propose in this paper, by contrast to most of the existing work on human capital acquisition in the trade literature, does not restrict endogenous human capital to be binary and therefore can account for simultaneaous sorting up and sorting down.

In modeling occupational output as tradable tasks, we recall Grossman and Rossi-Hansberg (2006), who propose a two-sector model of trade in tasks and focus on the welfare effects of outsourcing those tasks that can be carried out abroad. Also similar is the recent work by Jung and Mercenier (2008), who propose a model that features outsourcing of intermediates and endogenous human capital decisions. In several key respects, however, their approach is most similar to a two sector set-up in practice; as a consequence, skill upgrading in their framework is necessarily uni-directional as in the more traditional trade literature.

Finally, from a more technical standpoint, our continuum framework is reminiscent of Dornbusch, Fischer, and Samuelson (1977). More recently, Yeaple (2005), Costinot and Vogel (2009), Anderson (2009), and Helpman, Isthoki, and Redding (2008) also incorporate a continuum feature that, as here, can generate non-monotonic welfare consequences of trade. None of these models endogenize workers' human capital decisions, however. Workers' skill levels are taken as given, which necessarily obviates the potential role of educational institutions in shaping the interaction between trade and income distribution.

The remainder of the paper is structured as follows. In Section 2, we introduce the model, analyze the effects of trade under the small country assumption, and give the equilibrium conditions for the large country case. Section 3 adopts functional forms to illustrate further the inner workings of the model and to present the equilibrium characteristics of a two country case with non-monotonic skill change. In Section 4 we turn to the model's policy implications by introducing the possibility of education subsidies and tariffs. Section 5 concludes.

2 The Model

The Home country is populated by a continuum of heterogeneous agents with unit mass. Individual agents differ in their inherent ability levels, a, assumed to be distributed continuously over the unit interval with cumulative distribution function $F(a)$ and corresponding density function $f(a)$. Every agent is endowed with a single unit of labor, which is supplied inelastically to the labor market.

The economy produces a single homogeneous final good, Y, using constant returns to scale technology and a continuum of intermediate tasks (or products) $j \in [0,1]$, where j may be thought of as an index of the intermediate sectors' technological sophistication. Each intermediate sector uses a specialized type of labor and produces under constant returns and perfect competition. Productivity is assumed to be the same for all workers of an acquired skill type, regardless of the agent's inherent ability.[4] The final good serves as numeraire with price denoted by $p \equiv 1$. Finally, we choose units so that the real wage in sector j, measured in units of the final good Y, is simply the trading price of the relevant intermediate good/task and is denoted by $w(j)$.

In order to supply one unit of specialized labor of type j, agents have to acquire the required skills through training and education. The cost (in units of the numeraire, Y) to agent $a \in [0,1]$ of acquiring the skills for a given sector $j \in [0,1]$ is described by the function $c(j,a)$, which is twice continuously differentiable in each argument. We assume that the cost of skill acquisition is increasing in the technological sophistication of the sector and decreasing in the ability level of the agent; further, the marginal cost of upgrading skills from one sector to the next is lower for high ability agents;

[4] We could have built worker heterogeneity into productivity (and hence wages) rather than educational costs, as we do here. Going down that route would generate the same sorting of workers across sectors. In fact, there exists a one-to-one mapping between both approaches. We follow the approach chosen here as it lends itself more readily to policy analysis, a task we take up later in this paper.

finally, the cost of skill acquisition is convex across sectors for every agent. Formally, we make the following assumption on the cost of skill acquisition:

Assumption 1

$$\begin{aligned} \frac{\partial c(j,a)}{\partial j} &> 0, \qquad \frac{\partial c(j,a)}{\partial a} < 0 \\ \frac{\partial^2 c(j,a)}{\partial j \partial a} &< 0, \qquad \frac{\partial^2 c(j,a)}{\partial j^2} > 0. \end{aligned} \tag{2.1}$$

Additionally, in the interest of tractability, we will consider the following simplification:

$$c(j,a) \equiv h(a)g(j) \tag{2.2}$$

$$c^*(j,a) \equiv h(a)g^*(j), \tag{2.3}$$

where the functions $h(\cdot)$ and $g(\cdot)$ are twice continuously differentiable and non-negative over the unit interval. Note that Assumption 1 implies that $h'(a) < 0$, $g'(j) > 0$ and $g''(j) > 0$ for all $j \in [0,1]$.

Optimal Sorting and Production. Agents consume only the final good Y with non-satiated preferences. Thus, when deciding which sector to enter, every agent a chooses j to maximize his net real wage, $w(j) - c(j,a)$. Taking the wage schedule as given, the first order condition for each individual's optimal human capital level is then:

$$\frac{\partial c(j,a)}{\partial j} = \frac{dw(j)}{dj}, \tag{2.4}$$

Using superscript dots to denote derivatives with respect to j, the first order condition for agent a's optimal human capital decision/sectoral choice may be rewritten:

$$\dot{c}(j,a) = \dot{w}(j) = \dot{g}(j)h(a). \tag{2.5}$$

The second order condition of the agents' maximization problem is:

$$\ddot{w}(j) \leq \ddot{c}(j,a); \tag{2.6}$$

that is, the wage schedule — exogenous in a small open economy and endogenous if the country is large or autarkic — must be less convex than the cost function. Provided the second order condition is satisfied globally (i.e. for any $j \in [0,1]$), then in $\{j,a\}$ space, $\dot{c}(j,a)$ crosses $\dot{w}(j)$ at most once from below for any given a, and (2.5) implicitly defines a unique critical value of j for each agent. The first order condition in (2.5) thus determines the following allocation (or self-sorting) of ability types to sectors:

$$a(j) \equiv h^{-1}\left(\frac{\dot{w}(j)}{\dot{g}(j)}\right). \tag{2.7}$$

Note that the second inequality in Assumption 1 ensures that $h(\cdot)$ is invertible so that $a(j)$ is defined. When $a(j)$ is strictly monotonic, and thus itself invertible, we will denote the mapping of ability types to sectors by $j(a)$.

The assumptions made earlier ensure several important properties of the mapping function in (2.7). The third inequality of Assumption 1 ensures single crossing, so that agents map to sectors assortatively;[5] the sorting function is strictly monotonic if the second order condition holds with strict inequality. Further, assuming that $h(\cdot), g(\cdot) \in C^2$ in j implies that if $w(j)$ is continuously differentiable (twice continuously differentiable), then the mapping function $a(j)$ is itself continuous (continuously differentiable). Finally, if the wage schedule is twice continuously differentiable, $a'(j)$ will be everywhere finite. To summarize formally:

Lemma 2.1 *For a given strictly increasing wage schedule, $w(j)$:*

i) $a(j)$ is non-decreasing (strictly increasing) in j if $\ddot{w}(j) \leq \ddot{c}(j,a)$ ($\ddot{w}(j) < \ddot{c}(j,a)$) $\forall j, a$.

ii) $a(j)$ is continuous (continuously differentiable) in j if $w(j) \in C^1(C^2)$.

iii) $a'(j)$ is finite for all j if $w(j) \in C^2$.

[5]Note that $\dot{w}(j) > 0$ is a necessary condition for positive production in sector j.

Proof: See appendix.

Notice that the derivative of the mapping function, $a'(j)$, indicates the density of agents in any given sector j. Thus, zero employment in a range of sectors would be represented by a flat segment of the $a(j)$ mapping function in $\{j, a\}$ space. Part *i*) of the preceding lemma rules out the possibility of empty sectors if the wage schedule is everywhere less convex than the cost schedule for all agents and all sectors. By similar logic, a potential mass point in the distribution of agents across sectors would be represented by an infinite derivative of the $a(j)$ function. Part *iii*) indicates that continuity in the $\ddot{w}(j)$ schedule is sufficient to rule out the possibility of mass points, thus ensuring that the mapping of agents to sectors is strictly assortive: higher ability agents self select into *strictly* higher j occupations.

The sorting mechanism can best be illustrated by a pair of simple graphs. Figure 2 depicts the optimal sorting of agents to sectors as a function of the local wage schedule and the cost of education for each agent. We suppose for the sake of simplicity that the wage schedule is such that for every a there exists a unique optimal sectoral choice j, as depicted.[6] Panel A illustrates agents' optimal sectoral choice according to the first order condition in (2.5). Notice that the second order condition requires that $\dot{w}(j)$ crosses $\dot{c}(j, a)$ from above at the optimal occupation, $j(a)$. Further, the assumption that $\frac{\partial \dot{c}(j,a)}{\partial a} < 0$ ensures assortive matching of agents to sectors so that $j'(a) \geq 0$. Panel B depicts the resulting mapping of agents to occupational sectors.

To elucidate further the sorting mechanism, consider the comparative statics of an exogenous change in the domestic price/wage schedule (for instance, in a small open economy because of a shift in the terms of trade). From Panel A, if the intermediate price schedule is everywhere flatter than before, so that $\dot{w}(j)$ shifts down, every

[6] In principle, the derivative wage schedule could cross a given derivative cost function several times. In that instance, the globally optimal $j(a)$ would be an element (or subset) of the critical values defined by (2.7) for which the second order condition is also satisfied.

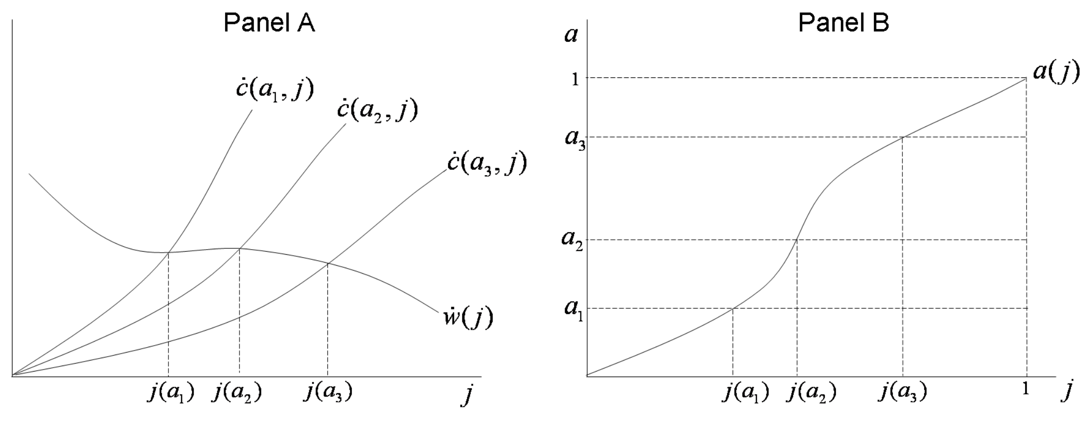

Figure 2: Optimal Sorting

agent will choose a lower j occupation, or 'sort down' in response to the decreased wage premia for skill upgrading.[7] Conversely, if the wage schedule is everywhere steeper than before, agents will choose higher skilled occupations, or 'sort up'. Note that for any vertical shift in the wage schedule for which the *derivative* wage schedule, $\dot{w}(j)$, remains unchanged, there will be no impact on agents' occupational choices or aggregate output. It is not the absolute *level* of wages, but rather the relative wage *differences* – reflected in the derivative wage schedule – that determine sorting. (Welfare is of course another matter.) The following proposition formalizes, assuming a parsimonious case of unique single crossing for all agents:

Proposition 2.1 *If the second order condition* (2.6) *holds with strict inequality for all* j*, then for any two wage schedules* $w_1(j)$ *and* $w_2(j)$*:*

i) if $\dot{w}_1(j_1(a)) = \dot{w}_2(j_1(a))$*, then agent* a *would choose the same sector under either wage schedule:* $j_1(a) = j_2(a)$*;*

[7] In a dynamic framework in which agents cannot recoup the costs of over-education (in essence reselling their degrees), we would expect agents to remain in their same jobs or, if $\dot{w}(j) < 0$ in the relevant region, to shift into lower-skill work for which they are then overqualified.

ii) if $\dot{w}_1(j_1(a)) > \dot{w}_2(j_1(a))$*, then agent a would choose a higher sector under* w_1 *than under* w_2*:* $j_1(a) > j_2(a)$*;*

iii) if $\dot{w}_1(j_1(a)) < \dot{w}_2(j_1(a))$*, then agent a would choose a lower sector under* w_1 *than under* w_2*:* $j_1(a) < j_2(a)$*.*

Proof: The result is immediate from the definition of $a(j)$ in (2.7).

Since we are interested in non-monotonic skill change – i.e. the simultaneous 'sorting up' and 'sorting down' of workers – note that the proposition directly implies the following:

Corollary 2.2 *Non-monotonic skill change occurs if and only if there exists a pair of sectors* $\{j_l, j_h\}$*, each in the unit interval, such that* $\dot{w}_1(j_l) < \dot{w}_2(j_l)$ *and* $\dot{w}_1(j_h) > \dot{w}_2(j_h)$*.*

That is, non-monotonic skill change will occur if (but only if) the two wage schedules cross at least once.[8]

Now that we have analyzed the sorting mechanism, we are in a position to close the model by defining appropriate equilibrium conditions. We will do so first for the case of a small open economy, before turning attention to the case of two large countries.

Equilibrium Conditions for a Small Open Economy. The domestic supply schedule for intermediates is given by the density of workers of each type in the

[8] The corollary continues to assume a case in which agents' second order condition holds globally, as stated in Proposition 2.1. For a more complex case in which there are multiple local solutions to the optimal sectoral choice problem for at least some agents, we cannot characterize the outcomes so simply. We do not find such technical possibilities particularly interesting or economically relevant, however.

population, multiplied by the density of workers in each sector:

$$y_s(j) = a'(j)f(a(j)). \tag{2.8}$$

Notice that the supply of intermediates depends on both the wage schedule and the cost of skill acquisition, via the $a(j)$ mapping function.

Based on the total supply of intermediates, domestic output of the final good is given by $Y \equiv \psi(\vec{y})$, where $\psi(\cdot)$ denotes the constant returns technology used to produce the final good and each $y(j) \equiv y_s(j) + y_t(j)$ includes net imports of intermediate products, $y_t(j)$.[9] We denote the total local unit factor demand for intermediate j by $x(j) \equiv x_j(\vec{w}, 1)$,[10] and note that in general it depends on the complete wage schedule.

The equilibrium conditions are then as follows. *Full employment* requires that the density of agents in each sector maps to the unit mass of population; i.e.:

$$\int_0^1 a'(j)f(a(j))dj = 1. \tag{2.9}$$

Market clearing for each intermediate implies:

$$y(j) = a'(j)f(a,j) + y_t(j) = x(j)Y \qquad \forall j \in [0,1]. \tag{2.10}$$

The *zero profit* condition in aggregate good production requires total revenue to equal total factor payments so that:

$$Y \equiv \psi(\vec{y}) = \int_0^1 w(j)[a'(j)f(a(j)) + y_t(j)]dj. \tag{2.11}$$

Finally consumers' *balanced budget* condition requires that consumption expenditure on the final good equals workers' net income:[11]

$$Y^c = \int_0^1 [w(j(a)) - c(a, j(a))]da, \tag{2.12}$$

[9] $y_t(j) < 0$ indicates a net export.

[10] Recall that constant returns to scale technology implies that conditional factor demand for each intermediate j may be written $x_j^T(\vec{w}, Y) = x_j(\vec{w}, 1)Y \equiv \arg\min_{x_j} \vec{w} \cdot \vec{x}$ s.t. $\psi(\vec{x}) \geq Y$.

[11] Tariff revenue, if appropriate, would simply be added to the RHS of (2.12).

where Y^c denotes aggregate consumption of the final good. Any trade imbalance in intermediates is made up by shipments of the final good Y.[12]

For a small open economy, the system described by (2.9) - (2.12) pins down, for a given wage schedule, the equilibrium allocation of agents to occupational sectors, intermediate production levels and trade, as well as aggregate final good output and consumption. For a large economy, on the other hand, the wage/intermediate price schedule is endogenous and depends on the rest of the world.

Equilibrium Conditions for Large Economies. The wage schedule becomes endogenous once we consider two large countries: Home, the country previously described, and Foreign (variables denoted by *). Let Foreign mirror Home in all aspects except the educational cost structure: Foreign has the same unit mass of population with an identical ability distribution, the same non-satiated preferences and inelastic labor supply, and the same production technology for intermediates and the final good. We restrict attention to differences in educational institutions because this is the novel force driving comparative advantage, and ultimately the focus of this paper.[13]

Under autarky each economy is characterized by a system of equilibrium conditions analogous to equations (2.9) - (2.12) above. Solving these, together with the autarkic condition that trade is zero ($\vec{y}_t = \vec{y}_t^* = \vec{0}$) yields the autarkic equilibrium wage schedules, which we denote by $w_A(j)$ for Home and $w_A^*(j)$ for Foreign. The equilibrium wage schedules in turn pin down all other variables of interest. Note that

[12] If Y is not tradeable, then balanced trade in intermediates requires: $\int_0^1 w(j)y_t(j)dj = 0$. (Moreover, if some intermediates are not traded in equilibrium (and Y is non-tradeable), then the price of the numeraire in Home may differ from the world market price of Y.)

[13] Since technology differences are the main focus of the seminal work by Dornbusch, Fischer, and Samuelson (1977), we silence that well understood mechanism in the baseline version of our model. One could easily introduce different intermediate good productivity in an extension of the basic model to study the interaction between technology, educational institutions, and trade.

the first order condition ensures that the more convex (in j) is the local educational cost function, the steeper the autarkic equilibrium wage schedule (all else equal). This makes sense; to the extent that skill upgrading becomes increasingly expensive for more sophisticated (high j) sectors, the higher the incremental wage increases must be to induce workers to enter the most demanding occupations.

Under free trade, intermediates can be traded on the world market. The set of world market clearing conditions for intermediates is therefore:

$$y(j) + y^*(j) = x(j)Y + x^*(j)Y^* \qquad \forall j \in [0,1]. \tag{2.13}$$

Additionally, we require that:

$$y_t(j) = -y_t^*(j) \qquad \forall j \in [0,1]. \tag{2.14}$$

That is, Home imports have to equal Foreign exports and vice-versa. The remaining equilibrium conditions – full employment, zero profit, and balanced budget in (2.9), (2.11), and (2.12) respectively – are the same as before, for both Home and Foreign. Together, these equilibrium conditions jointly determine the unified free trade equilibrium wage schedule, which we denote by $w_{FT}(j)$. This wage schedule then pins down the allocation of workers across sectors (and thus the distribution of human capital within and across countries), the supply of intermediates, aggregate output, and consumption in each country, as well as the pattern of trade. Note that for any sectors in which only one country produces in equilibrium, the market clearing condition in (2.13) has a single term on the left hand side, so that only the producing country's educational cost structure will influence the wage directly.

In general, the market clearing conditions can be characterized by a differential equation of the wage schedule over j. Collapsing the system of intermediate market clearing conditions (of which there are an uncountable infinity) to a single differential equation yields enormous returns in model tractability: namely, equilibrium properties can be summarized by the behavior of the wage schedule over $j \in [0,1]$ as above.

At the same time, however, given that the equilibrium wage schedule is the solution to a differential equation of the third order,[14] it should not be surprising that closed form solutions prove the exception rather than the rule.

The following section describes a case in which functional form assumptions offer clean analytical solutions in the general equilibrium model. In generating a set of closed form results, we highlight the role of educational institutions in determining both comparative advantage and the implications of trade for human capital acquisition, welfare, and income distribution within and across countries.

3 A General Equilibrium Example

In this section we provide a concrete example of our model that illustrates the simultaneous 'sorting up' and 'sorting down' of moderate ability agents and the negative welfare effects trade can have on the middle class. In order to make things tractable, we assume the following cost structures of education in home and foreign respectively:[15]

$$c[j,a] = \frac{(1-a)}{a} * \frac{2j^2}{5} \tag{3.1}$$

$$c^*[j,a] = \frac{(1-a)}{a} * \frac{2j^3}{3} \tag{3.2}$$

The crucial feature of the above functional forms lies in the relative convexity of both educational cost structures: the foreign educational cost structure is more

[14] $y(j)$ depends on $a'(j)$, which is a function of $\dot{w}(j)$ and $\ddot{w}(j)$ while $x(j)$ depends in general on the complete $w(j)$ schedule. Moreover, in a model with multiple final goods in which aggregate final goods output depends on its own price (i.e. $Y(p)$), the market clearing condition would instead be given by fourth order differential equation.

[15] We adopt these functional forms primarily for analytical convenience, but also because they provide a 'nice' example in which both countries produce all goods. Appendix A1.2 explores modified functional forms designed to deliver a case of "limited diversification" in which one country is the exclusive producer of some goods in equilibrium.

convex in j than is the domestic cost structure. Formally, we need the $\dot{c}^*(j, a)$ and $\dot{c}(j, a)$ functions to cross somewhere in the unit interval for at least some ability levels, $a \in [0, 1]$. From a technical standpoint, the relative convexity of the Foreign cost structure ensures that Foreign will have comparative advantage in some middle j sectors (necessary to generate non-monotonic skill change at Home following an opening to trade).

In practice, our assumption over the relative convexity of the cost schedules is consistent with a faster increase in the marginal cost of education abroad, as individuals move up the skill acquisition ladder. We find this assumption particularly plausible in the context of many developing countries, where barriers to education make the shift from secondary to university to graduate education increasingly difficult relative to the industrialized world. Notice that it is only the relative costs of increasing education — i.e. the derivative of the wage schedules — that matter for sorting. Educational cost levels could be higher across the board in Foreign than in Home (which again seems particularly plausible in developing vs. developed country comparisons) without changing our results, apart from the quantitative welfare levels.[16] We should note, too, that the model would easily permit alternative cost structures; we focus on this case because we are most interested in the potential for non-montonic skill change resulting from trade.

On the factor demand side, we assume Leontief production of the final good, thereby abstracting from possible substitution effects across intermediates.[17] With $\psi(\vec{y}) \equiv \min\{y_0, ..., y_1\}$, unit factor demand is simply one in each sector and country,

[16]To verify, simply add a constant to $c^*[j, a]$) and note that the sorting mechanism will be unchanged.

[17]More generally, substitution effects would dampen the magnitude of wage schedule changes, but would not overturn our qualitative findings; the technical benefit of the Leontief assumption is that $x_j(\vec{w}, 1) = 1 \forall j$ so that the intermediates market clearing condition is only a second order differential equation (rather than third order) with a closed form solution.

regardless of the wage schedule; thus, $x(j) \equiv x_j(\vec{w}, 1) = x^*(j) = 1$. Following the solution procedure outlined in the previous section, we solve for the equilibrium wage schedules:

$$\dot{w}_A = \frac{4(1-j)}{5}, \tag{3.3}$$

$$\dot{w}_A^* = 2j - 2j^2, \tag{3.4}$$

$$\dot{w}_{FT} = \frac{j(2+j-10j^2) + \sqrt{j^2(4+j(4+4j(121+20j(-9+5j))))}}{10j}. \tag{3.5}$$

Where we have used the boundary condition that the wage schedule must be flat at the upper end, $\dot{w}(1) = 0$, to pin down the respective constants of integration.[18] Figure 3 shows that the slope of the equilibrium wage schedule under free trade is a weighted average of the autarky values. Note in particular that the intersection of the autarky wage slopes leads to the same value of the slope of the equilibrium wage schedule under free trade.

As discussed earlier, the equilibrium wage schedule implies a corresponding mapping of agents to sectors by ability level. In autarky, of course, the Leontief technology assumption implies a uniform density of workers across sectors:

$$a_A(j) = a_A^*(j) = j. \tag{3.6}$$

Under free trade the sectoral mappings take a polynomial form (we omit the functional forms here for brevity), depicted in Figure 4. Home's free trade $a(j)$ mapping is in pink, Foreign's free trade mapping is in dark blue, and the autarkic mapping schedules for both Home and Foreign are represented by the light blue diagonal on the 45 degree line.

Figure 4 illustrates the reallocation of agents brought about by trade liberalization. Where the free trade mapping function lies above the diagonal, the corresponding ability level self-selects into a lower j sector following liberalization; i.e. agents

[18] This boundary condition ensures that there is not a mass of workers clustered in sector $j = 1$.

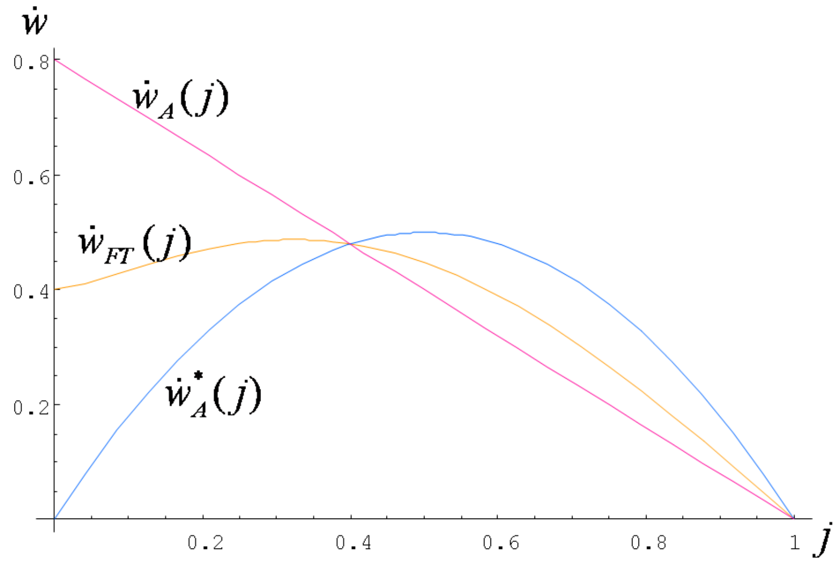

Figure 3: Slope of the Equilibrium Wage Schedules

have sorted down. Where the free trade mapping function lies below the forty-five degree line, agents self select into higher j occupations and human capital levels following the opening of trade. Overall, we see that in Home agents in the lower portion of the population distribution shift to lower j sectors, while agents above $a = .4$ shift up, thus vacating the middle j sectors toward the skill-acquisition extremes. The effects in Foreign are simply the reverse, since total labor supply of each factor must add to 2 under the Leontief final goods production structure.

Figure 5 depicts the resulting shift in employment density across sectors, which is equivalent to the supply of each intermediate output (given our assumption that $a \sim U[0, 1]$). Again, note that the Leontief technology ensures uniform employment distribution in autarky. We again see that trade pushes Home workers to the skill acquisition extremes, while 'middle class' employment flourishes in Foreign post liberalization.

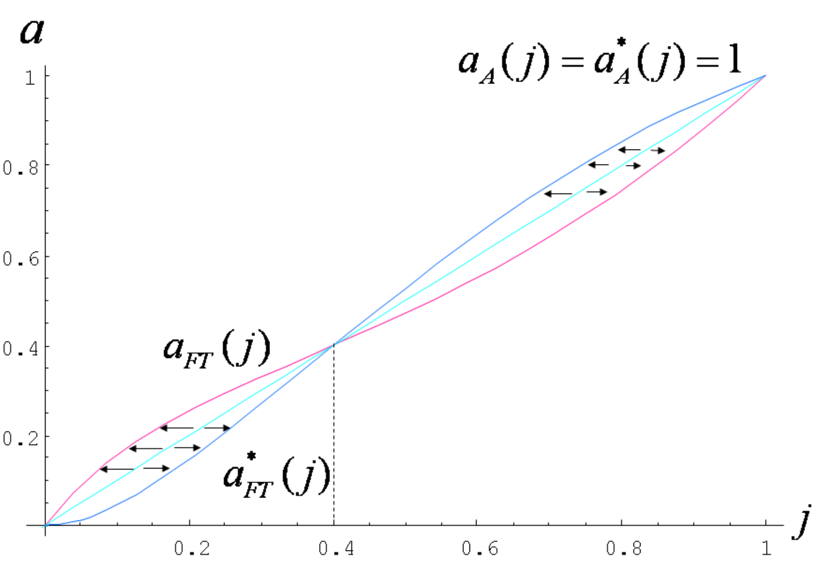

Figure 4: Mappings $a_{FT}(j)$ and $a^*_{FT}(j)$.

In addition to the employment and skill acquisition effects of trade liberalization, we are of course keenly interested in the welfare implications of freer trade. To get at real welfare effects, we first determine the real wage schedules under free trade and autarky using the zero profit condition for final goods production[19] to pin down the equilibrium wage level. The equilibrium real wage in a given sector $\hat{j}$ is given by $w_0 + \int_0^{\hat{j}} \dot{w}(j)dj$, where the base wage in sector $j = 0$ is determined by $w_0 \equiv 1 - \int_0^1 \dot{w}(j)dj$. Solving, we find for low and high j sectors, the wage schedule increases at Home and decreases in Foreign country following trade liberalization, while the converse holds for mid-range sectors.

In what follows we first focus on the welfare effects in Home. As we will see below, the effects in Foreign are virtually a mirror image. The real welfare change consists of the effect on the real wage and the change in the realized cost of education.

[19]Under the Leontief production structure and choice of Y as numeraire, zero profit implies $1 = \int_0^1 w(j)dj$.

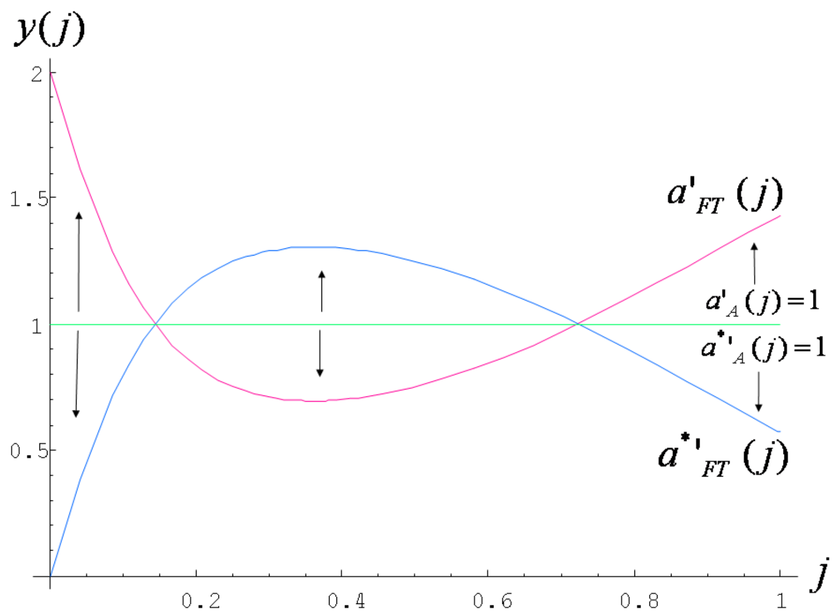

Figure 5: Employment Density by Sector: Autarky and Free Trade.

We analyze first the effect on the real wage. The two panels in Figure 6 depict respectively the change in the real wage in sector j and the change in the real wage of agent a given her optimal sectoral choice under each trading regime.

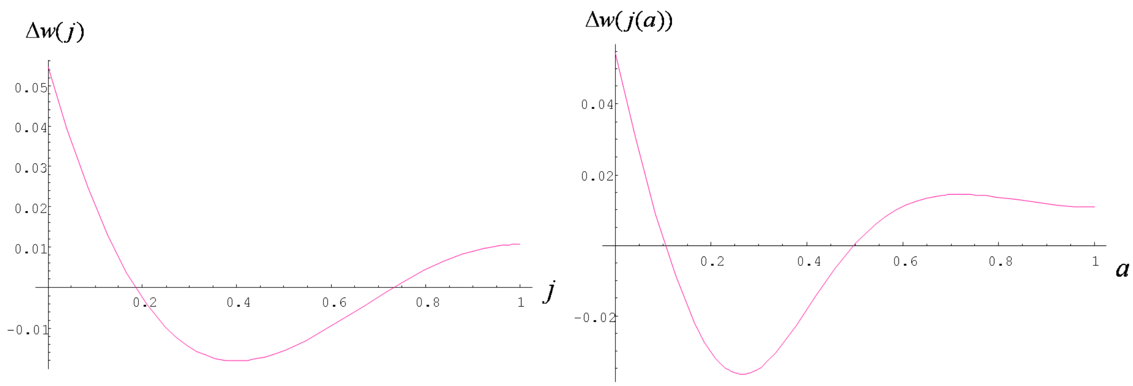

Figure 6: Effect of Trade on Wages at Home.

From the first panel in Figure 6, we see that real wages rise for the low and high j sectors, and fall for sectors j between about .2 and .7. The second panel takes into account the induced occupational shift, confirming that the change in realized real wages is non-monotonic across workers: agents with low ability earn higher real wages under trade, agents with high ability do as well, and agents in the lower-middle portion of the ability distribution see their real wages fall.

Figure 7 shows the change in the real cost of education across workers. Remembering that agents in the lower forty percent of the ability distribution optimally sort down while agents in the upper part of the distribution choose to sort up, it is obvious that the real cost of education should decrease for the left portion of the distribution and increase for the right.

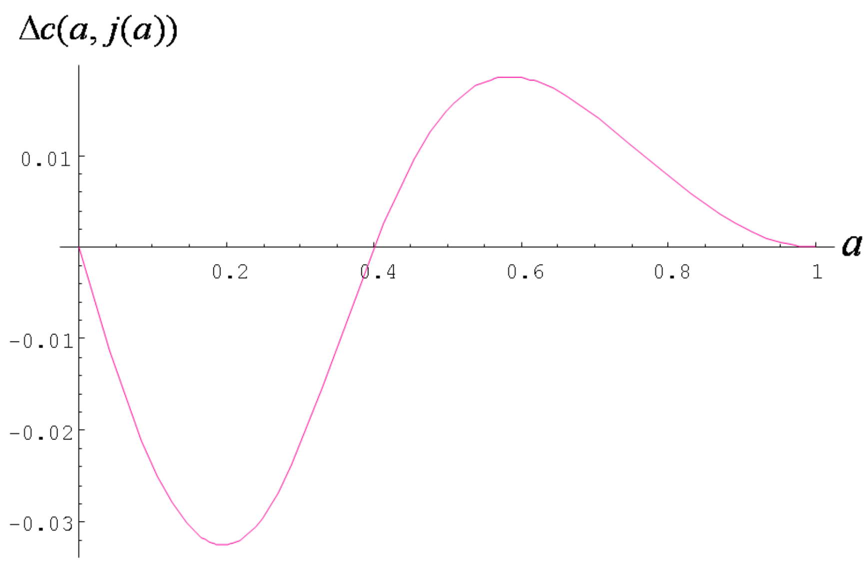

Figure 7: Change in the Home Real Cost of Education across Workers

Figure 8 shows the net welfare change for Home's population. Combined with the effect on the real wage, the adjustment for the changing cost of education has shifted the identity of the 'biggest loser' to the right. Indeed, consider the plight of the agent $a = .6$. Although her real wage has increased, the increased cost of education required to achieve the higher paying job more than offsets the wage gain so that the net welfare change is negative. Conversely, agent $a = .2$ suffers a substantial real wage loss yet enjoys a modest net welfare improvement due to his now lower cost of education. A crucial caveat to this second statement is that lower costs of education cannot be recovered if they are sunk. In a dynamic framework with unanticipated trade shocks, we therefore would expect to see the burden of increased costs of (potentially mid-career) education manifest in net welfare changes, while education savings would not be realized for the older generations.

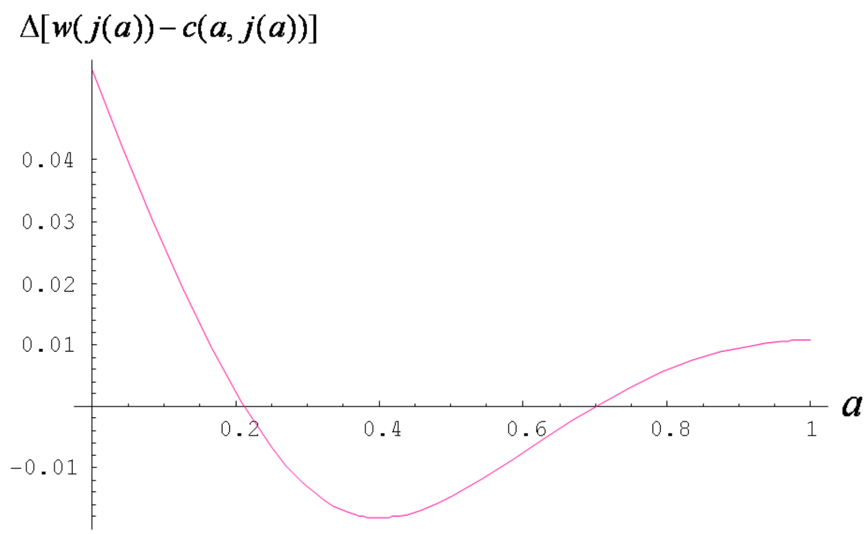

Figure 8: Net Welfare Effect of Trade in the Home Country.

Turning now to the Foreign country, we find that the net welfare effects are a mirror image of what happens at home. In contrast to Home, the real wage increases most for Foreign agents in the middle of the income distribution while falling at the distribution extremes. At the same time, the cost of education rises for the lower forty percent of the distribution of workers and fall for the remainder. The net welfare effect of trade is shown in the final panel of Figure 9, where we see that middle ability agents gain from trade, while the highest and lowest ends of the population distribution lose.

To summarize the results from this general equilibrium example, we depict both the Home and Foreign net welfare changes by worker in Figure 9. While in the Home country it is the medium ability agents who suffer, their Foreign counterparts are the main beneficiaries of trade liberalization, together with high and low ability agents at Home. Integrating the net real effects in each country confirms that there are positive gains from trade for both countries.

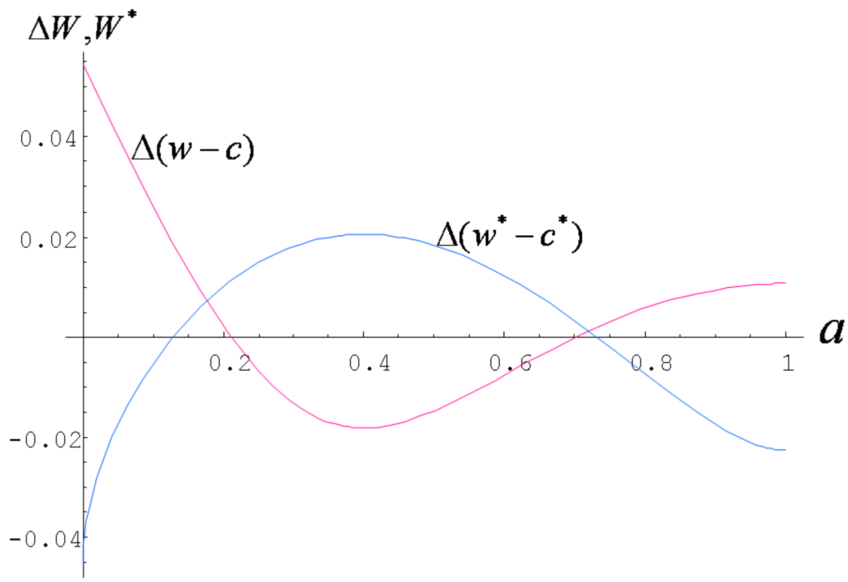

Figure 9: Opposing Individual Welfare Effect of Trade in Home and Foreign.

4 Policy Implications

Our model immediately suggests itself to policy analysis, as indeed was our initial intention. In the next section, we focus on the potential role of educational policy and trade protection in shaping the distribution of income and human capital, and note in particular the extent to which the two policy instruments are complementary.

4.1 Educational Policy

In practice, educational policy takes many forms. Broadly speaking, educational initiatives can increase educational productivity (for instance, by reducing institutional overhead, through curriculum reform, or by expanding teacher education and incentives), decrease the costs of education borne by students, or both. Here, we focus on the latter as it is simpler from a modelling perspective and largely isomorphic in its underlying implication for human capital decisions.[20] We define an educational subsidy, $s(j)$, as a means to help individuals acquiring education in a given sector/task j shoulder the monetary cost of doing so, $c(j, a)$. By assumption, the subsidy can vary by type of education or skill level to be acquired, but cannot be conditioned on the unobservable inherent ability level of the agent, which is private information. In order to balance the government's budget, we assume that the subsidies are financed by a poll tax.

Recall the first order condition of the educational choice problem derived earlier in (2.5). Introducing an educational subsidy leads to the following augmented first order condition:

$$\dot{c}(j, a) - \dot{s}(j) = \dot{w}(j), \tag{4.1}$$

where $\dot{s}(j)$ is the first derivative of the subsidy function with respect to j (assuming $s(\cdot)$ is differentiable). We assume that the educational subsidy function is such that the left hand side of the augmented first order condition in (4.1) satisfies the derivative property assumptions (convexity, single crossing) we made regarding the educational cost function in (2.1).

[20]Improving the quality of education would have the same effect on individuals' skill acquisition incentives as reducing the cost of education, but the former would simultaneously increase worker productivity (conditional on education level) while the latter would not.

For the following result, we adopt the small country assumption to rule out any effects on the world price schedule. Moreover, we continue to assume a 'nice' case in which both before and after the introduction of the educational subsidy, the second order condition for each agents' optimal skill acquisition decision is satisfied globally, so that the derivative cost schedule crosses the derivative wage schedule at most once from below. The following proposition then follows directly from the augmented first order condition in (4.1):

Proposition 4.1 *If* $c(j,a)+s(j)$ *satisfies properties analogous to* (2.1)*, then for all* $j \in (0,1)$ *the educational subsidy schedule* $s(j)$ *has the following (local) effects:*

i) if $\dot{s}(j)=0$ *then* $a(j,\vec{s})=a(j,\vec{s}=0)$*;*

ii) if $\dot{s}(j)>0$ *then* $a(j,\vec{s})\leq a(j,\vec{s}=0)$*;*

iii) if $\dot{s}(j)<0$ *then* $a(j,\vec{s})\geq a(j,\vec{s}=0)$.

The first part of the proposition implies that a uniform subsidy would not affect individuals' educational choices; rather, it is the *marginal* cost of additional education that matters.[21] Parts *ii*) and *iii*) imply that if the subsidy scheme is uniformly progressive (regressive) – offering higher subsidies at lower (higher) education levels so that $\dot{s}<0(>0)$ across the board– then all agents would sort monotonically into higher (lower) skilled occupations. More generally, the local effect of a subsidy change on agents initially located in a particular sector j would be to induce educational upgrading (downgrading) if the net marginal cost of education is decreased (increased) by the subsidy scheme.

Now consider the following: how might the government use educational policy

[21]This finding is little surprise as a uniform subsidy to agents in all sectors simply cancels the poll tax (given full employment and positive education subsidies for all sectors $j \in [0,1]$).

to soften the impact of globalization?[22] The negative impact of trade — at least in the example discussed earlier — is felt by middle-ability agents. If the government, perhaps in the interest of political stability that relies on a sizable middle class, wants to counteract the 'vanishing middle class' phenomenon, it has to provide an educational subsidy schedule that features a positive slope for low ability agents and a negative (or at least much flatter) slope for higher ability agents. In other words, it has to channel educational subsidies towards mid-level skill acquisition. The German educational system and the country's strong middle class can be regarded as anecdotal evidence for such an approach. In contrast, educational tax credits, which are roughly proportional to the cost of education, are regressive in the sense that they provide little to no (additional) subsidy through the secondary school level, and offer increasing subsidies thereafter.[23]

Notice that targeted education subsidies toward middle class workers effectively constitute a production subsidy to import-competing sectors. From an efficiency point of view, such a policy is clearly counter-productive, as redistribution could be more efficiently achieved through more direct means. If, however, the country is large enough to influence the world market price schedule, then targeted middle class education subsidies would also improve the terms-of-trade and thereby shift part of the efficiency cost of middle class education subsidies onto foreign competitors. That said, a favorable shift in the terms-of-trade can be more efficiently achieved by trade policy than through educational subsidies, as we discuss in the next sub-section.

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[22]This is not to say that we advocate such an objective from an efficiency point of view; rather, we simply explore the possibility as one that seems consistent with many politicians' stated goals.

[23]Although per-annum education credits are generally capped (making the effective subsidy uniform for sufficiently high tuition levels in a given year), the lifetime educational tax credit is clearly proportional to years of schooling, and thus (generally) increasing with skill acquisition.

4.2 Trade Policy

We offer a brief positive analysis of how trade policy influences educational choices, before turning to the normative question of optimal instrument choice. Let us initially adopt the small country assumption and define the specific tariff/export subsidy for good j by $t(j)$, so that the net domestic price/wage is now given by $w^d(j) \equiv w(j) + t(j)$, where $w(j)$ denotes the world price of good/task j.

On the supply side, the effect of a specific tariff (export subsidy) on human capital decisions is virtually identical to that of an educational subsidy. This is readily apparent from the augmented first order condition of the individuals' educational decision, which now takes the form:

$$\dot{c}(j,a) = \dot{w}(j) + \dot{t}(j),$$

where $\dot{t}(j)$ is the derivative of the trade tax/subsidy schedule with respect to j, and we assume that $c(j,a) - t(j)$ satisfies properties analogous to (2.1). As with the education subsidy, a uniform trade tax schedule across all tasks will have no effect on agents' human capital decisions.[24] If the trade tax/subsidy schedule exhibits positive or negative slope, on the other hand, then it affects the individuals' skill acquisition decision: a positive (negative) slope induces individuals to choose a higher (lower) type of education.[25]

Now let us suppose that a policymaker seeks to mitigate the impact of globalization on the middle class by using tariffs. To be consistent with commonly observed policy measures and WTO rules, we consider the effect of imposing tariffs only on

[24] An ad-valorem tax/subsidy schedule would have an effect, of course, as its specific equivalent would imply a higher net wage derivative schedule for more skilled (higher wage) sectors.

[25] Given the immediate parallel with the case of an educational subsidy (which subtracts $\dot{s}(j)$ from the left hand side of the agents' first order condition rather than adding $\dot{t}(j)$ to the right hand side), we omit restating the results as a formal proposition. Simply note that the trade tax/subsidy result can be read directly from Proposition 4.1 replacing $s(j)$ with $t(j)$.

imported goods, without introducing trade policy on the export side. In particular, we consider import tariffs that are chosen in such a way as to 'close the lens' between the autarkic and the free trade wage schedules, as is shown in panel A of Figure 4.2. Clearly, these tariffs also affect the derivative wage schedule. Jumps occur at the thresholds j^* and j^{**}, that separate exported from imported intermediates, as depicted in Panel B of Figure 4.2. These discontinuities in the derivative wage schedule induce two 'empty' regions, as agents on the left side of those regions sort down and those on the right side sort up.[26] These 'empty' sectors have no domestic employment, and are thus Ricardian-like regions in which the Foreign country would be the sole producer worldwide, in line with the Dornbusch, Fischer, and Samuelson (1977) framework. Substantial levels of tariff protection could thus have the (unintended) consequence of dismantling some previously export-competing industries just below and above the lower and upper discontinuities.

It is clear that import protection will shelter the middle class from global competition in mid-range sectors, and may thus serve a policymaker's goal of influencing human capital decisions and the pattern of employment in the desired way. At the same time, however, tariffs also impose demand-side distortions that targeted educational subsidies would not.[27] If the country is large, this demand-side distortion will improve the terms-of-trade (by decreasing world relative demand for mid-range goods), whereas for a small country, any demand-side distortion constitutes a pure efficiency loss.

[26] The extent to which this happens clearly depends on the height of the jumps which is determined by the tariff schedule.

[27] The exception is the case of Leontief final good production, which entails no distortion. However, this is clearly a special case and unique to the Leontief functional form assumption.

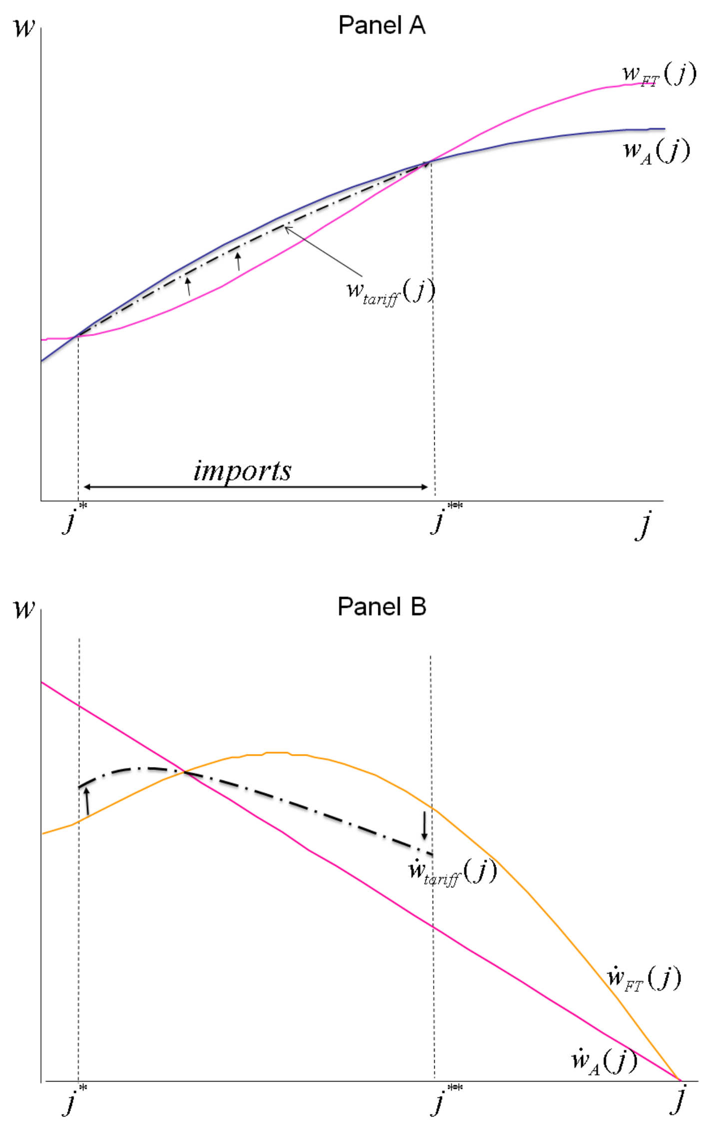

Figure 10: Effects of an import tariff

4.3 Instrument Choice

Moving to a normative comparison of different policy instruments, it is clear that the first best policy instrument depends on the government's underlying objective. If the government wants to shift the distribution of income, then of course lump sum taxes are the first best policy choice; if instead the goal is to shift the distribution of human capital, then educational subsidies (or taxes) are most efficient; and if the government simply seeks to manipulate the terms of trade, then of course trade policy remains the best means to achieve that end. Summarizing another way, education subsidies constitute a first best policy for shifting the distribution of human capital, a second best policy for shifting income (which is more efficiency achieved through direct redistribution and lump sum taxation), and a third best policy for manipulating world prices (which is better done via import tariffs). Trade taxes/subsidies, on the other hand, are first best for manipulating the terms of trade, second best for manipulating educational decisions (which is better achieved through education policy), and worst for shifting the distribution of income. Regrettably, political rhetoric rarely seems to square with these efficiency rankings across policy instruments.

5 Conclusion

In this paper we develop a model of trade and education that allows for differentiated effects of trade liberalization on skill acquisition. Agents of different ability levels self-select into sectors by acquiring the specific education necessary to work in a particular sector, or to perform a particular task. This mapping of ability level to sectors depends on the wage or price schedule and hence on a country's openness to trade. We show how changes in the price schedule affect this mapping and lead agents to sort up and down simultaneously. If a country's educational cost structure is less convex than that of its trading partners, then low ability agents sort down and higher ability agents sort up, and we obtain a 'vacating of the middle' with corresponding negative welfare effects for the middle class. This result provides one possible explanation for the current public concern over the negative effects of globalization for 'average' workers.

Our framework can shed light on the potential differential impacts of strengthening educational institutions. Government subsidies to education or similar institutional improvements that decrease the cost of skill acquisition over some ranges of sectors or for certain agents would impact the distribution of human capital decisions and thus the pattern of trade and comparative advantage, aggregate social welfare, and intra-national income distribution. In more general terms, the model developed here provides a novel reason for trade. By abstracting from differences in technology or preferences, we show how differences in educational institutions endogenously give rise to comparative advantage and hence trade.

Going forward, our model provides a novel framework with which to analyze the aggregate effects of targeted educational policies at the primary, secondary, or tertiary levels. Uniform 'across the board' subsidies to education are hardly the optimal policy recommendation according to our initial findings; perhaps highly focussed educational policies such as Brazil's are not as ill founded as some have suggested. Our model is similarly well suited to study the effects of educational migration, including the increased graduate attainment in the U.S. by foreign born students recently documented in Blanchard, Bound, and Turner (2008). Finally, In a somewhat more technical extension, one could explore systematically the nature of the interaction of technological changes with trade and education, in an effort to inform an empirical strategy for identifying the welfare effects of trade apart from technological innovation, while explicitly recognizing the endogeneity of worker's human capital decisions.

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A1 Appendix

A1.1 Proof of Lemma 2.1

1. Taking the derivative of $a(j)$ with respect to j yields:

$$a'(j) = \frac{1}{h'\left(\frac{\dot{w}}{\dot{g}}\right)}\left[\frac{\ddot{w}\dot{g} - \dot{w}\ddot{g}}{\dot{g}^2}\right]. \tag{A1.1}$$

Substituting from the first order condition in (2.5):

$$a'(j) = \frac{1}{h'\left(\frac{\dot{w}}{\dot{g}}\right)}\left[\frac{\ddot{w}\dot{g} - h(a)\dot{g}\ddot{g}}{\dot{g}^2}\right]. \tag{A1.2}$$

Then, from the definition of the cost function:

$$a'(j) = \frac{1}{h'\left(\frac{\dot{w}}{\dot{g}}\right)}\left[\frac{\dot{g}(\ddot{w} - \ddot{c})}{\dot{g}^2}\right] \geq 0, \tag{A1.3}$$

using the second order condition ($\ddot{c} \geq \ddot{w}$) and the assumptions on the cost function in (2.1), which imply that $\dot{g} > 0$ and $h'(x) < 0$ iff $x > 0$. By assumption, $\dot{w} \geq 0 \forall j$, so

$$a'(j) = \underbrace{\frac{1}{h'\left(\frac{\dot{w}}{\dot{g}}\right)}}_{(-)} \left[\frac{\dot{g}(\ddot{w} - \ddot{c})}{\dot{g}^2}\right] > 0 \iff \ddot{c} > \ddot{w}. \diamond \tag{A1.4}$$

2. From the definition of $a(j)$:

$$a(j) = h^{-1}\left(\frac{\dot{w}}{\dot{g}}\right). \tag{A1.5}$$

Both $g(\cdot)$ and $h(\cdot)$ are twice continuously differentiable and invertible by assumption. Thus, $a(j)$ is continuous in j if $\dot{w}(j)$ is continuous (i.e. $w(j) \in C^1$). Moreover, $a(j)$ is continuously differentiable in j if $\ddot{w}(j)$ is continuous (i.e. $w(j) \in C^2$).

3. From part (1) above:

$$a'(j) = \frac{1}{h'\left(\frac{\dot{w}}{\dot{g}}\right)} \left[\frac{\ddot{w}\dot{g} - \dot{w}\ddot{g}}{\dot{g}^2}\right]. \tag{A1.6}$$

Again from our earlier assumptions, $h'(\cdot)$, $\dot{g}$, and $\ddot{g}$ are finite, and $\dot{g} > 0$. Thus, $a'(j) < \infty$ if and only if $\ddot{w}\dot{g} - \dot{w}\ddot{g} < \infty$. A sufficient condition is $w(j) \in C^2$, as stated in the lemma.

A1.2 GE Example Extension: Limited Diversification

In the baseline example, every task is carried out in both countries in equilibrium; that is, even under free trade, the production of intermediates remains fully diversified across the entire range of occupations. This needn't hold in general. We now present a modified case where trade induces one trading partner to stop production of a particular subset of intermediates, a phenomenon that is certainly relevant in reality, where for example certain inputs can be sourced locally only at very high cost. Suppose the educational costs take the following (slightly modified) form

$$c[j,a] = \frac{1}{a} * \frac{j^2}{2} \quad \text{(A1.7)}$$

$$c^*[j,a] = \frac{1}{a} * \frac{2j^3}{3} \quad \text{(A1.8)}$$

and assume the same production technology for the final good as before. We can solve for the autarky and free trade equilibrium wage schedules:

$$\dot{w}_A = 1, \quad \text{(A1.9)}$$

$$\dot{w}_A^* = 2j, \quad \text{(A1.10)}$$

$$\dot{w}_{FT} = \begin{cases} \frac{j+2j^2}{2j} & j < \frac{1}{4}(1+\sqrt{5}) \\ \frac{j}{2j-1} & j \geq \frac{1}{4}(1+\sqrt{5}) \end{cases} \quad \text{(A1.11)}$$

These are graphically depicted in Figure 11. Note that the free trade equilibrium slope of the wage schedule (sandwiched between the autarky schedules) consists of two parts: up to $j = \frac{1}{4}(1+\sqrt{5}) \equiv \bar{j}$ both countries produce each task, whereas above only the home country does. This is because at the upper bound ($j = 1$) the marginal costs of education for the most able agents in both countries differ: the domestic agent with $a = 1$ only has half as high a marginal cost as the foreign agent, $a^* = 1$. Hence in equilibrium, the most able foreign agent does not find it worthwhile to acquire the sophisticated skills necessary to carry out the most sophisticated task. Instead she chooses $\bar{j} < 1$.[28]

As before, graphing the ability-to-task mappings under autarky and free trade in Figure 12 indicates how trade liberalization affects the skill acquisition decisions: to the left of the intersection agents at home sort down, whereas higher ability agents sort up and acquire more sophisticated skills, and the opposite happens in the foreign country. Thus the result that domestic agents vacate the middle obtains here as well.

[28]We find this threshold by using the boundary condition $\dot{c}(a = 1, j = 1) = \dot{w}$ to determine the constant of integration of the w-dot schedule that equates home supply to world demand. Since 'smooth pasting' of the wage schedules for the upper and lower ranges implies the same derivative at the threshold, we find the threshold by plugging $a^* = 1$ into the 'only home production' schedule already determined. 'Smooth pasting' then pins down the constant of integration of the schedule to the left where both countries produce.

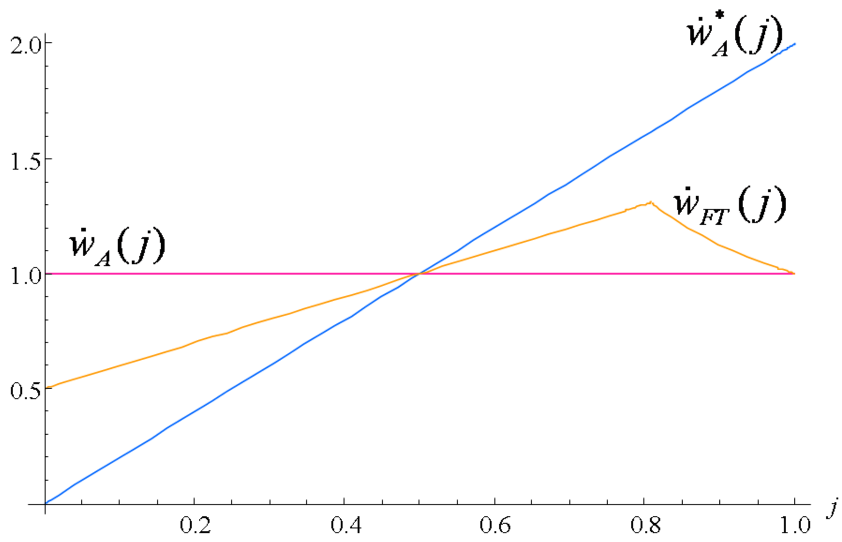

Figure 11: Wage schedules under autarky and free trade.

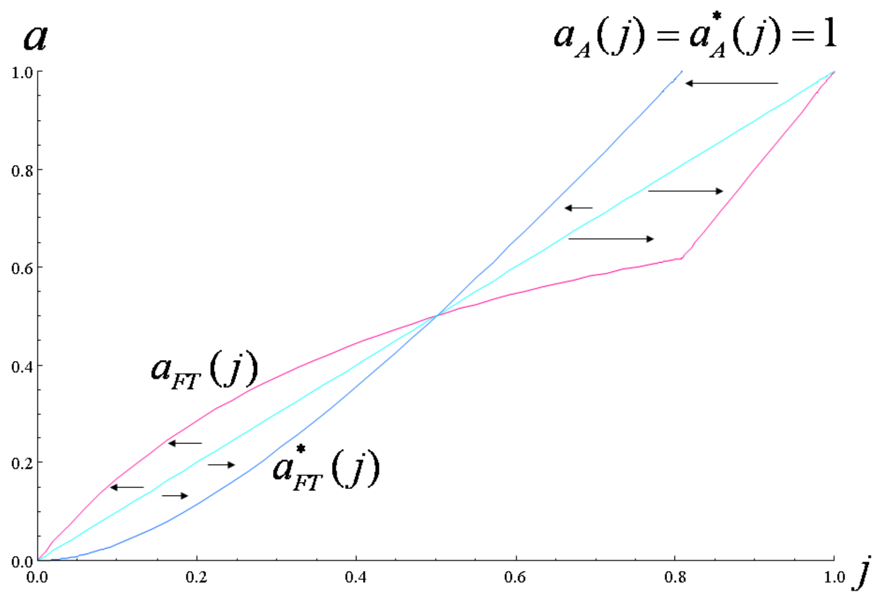

Figure 12: Ability-sector mappings under autarky and free trade.

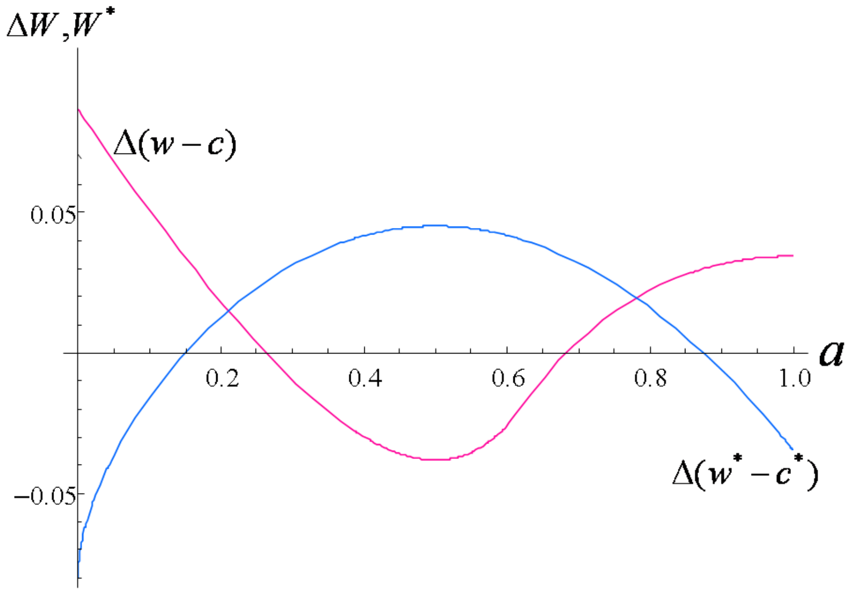

Figure 13: Net Welfare Effect of Trade in the Home Country.

Finally, comparing the wage change and the change in the cost of education gives the welfare effects depicted in Figure 13. Again we see that the middle ability agents at home lose out whereas agents at the bottom and top benefit from trade, and the welfare effects in the foreign country are the opposite. We have therefore confirmed that our previous result is robust to the possibility that countries restrict the range of tasks they produce in response to trade liberalization.