Handout 6: Hints to exercise 1 and 5

Exercise 1

a. The Agents have identical preferences, and the Egdgeworth box is square, the indifference curves are mirror images.

Since MRS^A=MRS^B=$\frac{x_2}{x_1}$, they will be equal with value one along the line $x_2 = x_1$. The contract curve is therefore the line $x_2 = x_1$

c.

- **i.** Method 1: The two agents' indifference curves are tangent when MRS^A=MRS^B=$\frac{p_1}{p_2}$
- **ii.** Method 2: Compute the demand functions and calculate the market clearing condition for one of the 2 goods.

$$D_1^A = \frac{(p_1 e_1^A + p_2 e_2^A)}{2p_1}$$

$$D_1^B = \frac{(p_1 e_1^B + p_2 e_2^B)}{2p_1}$$

$$D_1^A + D_2^B = 10$$

$$\frac{(2p_1 + 8p_2)}{2p_1} + \frac{(8p_1 + 2p_2)}{2p_1} = 10$$

$$\frac{p_1}{p_2} = 1$$

Exercise 5

a. On the Demand Side:

$$Max\, U(x,l) = \ln x + \ln(24 - l)$$

$$st.\ px = wl + \Pi$$

Set the Lagrangian and derive wrt x, l, and λ,

$$MRS = -\frac{w}{p} = -\frac{x}{(24 - l)}$$

$$\Rightarrow \begin{cases} l^s = 12 - \dfrac{\Pi}{2w} \\ x^d = 12\dfrac{w}{p} - \dfrac{\Pi}{2p} \end{cases}$$

On the Supply Side:

$$\Pi = py - wl$$

$\Pi = (p - w)l$ with $y = f(l) = l$

3 cases:

- $w > p$: The company make a loss on every unit so that it chooses to produce nothing. $y = l^d = \Pi = 0$
- $p > w$: The company make a positive profit on every unit produce so that it wants to produce more and more. $y = l^d = \Pi \to \infty$
- $p = w$: The company does not care how much it produces as profit is always equal to zero. $y = l^d \in [0, \infty); \Pi = 0$

b. The first case cannot be an equilibrium since it is not possible to get the consumer to supply $l^s = 0$ and to demand $x^d = 0$. The second case is no equilibrium either since the consumer cannot provide an infinite amount of labor. That leaves $\frac{w}{p} = 1$ as the only equilibrium. At this price and $\Pi = 0$, we get:

$$l^* = l^s = l^d = 12$$

$$x^* = x^s = x^d = 12$$