Handout 3 and 4: Hints

Exercises H3 a)

(This is one solution among many!)

$$MaxU(x,y) = \sqrt{x-\alpha}\,\sqrt{x-\beta}$$

$$s.t\ \ I = p_x x + p_y y$$

Set the Lagrange

$$£ = \sqrt{x-\alpha}\,\sqrt{x-\beta} - \lambda(p_x x + p_y y - I)$$

and derive with repsect to x, y, and I. The MRS is as follow:

$$\frac{y-\beta}{x-\alpha} = \frac{p_x}{p_y}$$

Substitute in the Budget constraint to obtain uncompensated demand functions D_x and D_y:

$$D_x(p_x, p_y, I) = \frac{I + p_x\alpha - p_y\beta}{2p_x}$$

$$D_x(p_x, p_y, I) = \frac{I + p_y\beta - p_x\alpha}{2p_y}$$

Plug these unconpensated demands into the Utility function to get the Indirect Utility fonction V:

$$V(p_x, p_y, I) = \sqrt{\frac{I + p_x\alpha - p_y\beta}{2p_x} - \alpha}\,\sqrt{\frac{I + p_y\beta - p_x\alpha}{2p_y} - \beta}$$

$$V(p_x, p_y, I) = \frac{I - p_x\alpha - p_y\beta}{2\sqrt{p_x p_y}}$$

Reverse to obtain the Expenditure function E:

$$E(p_x, p_y, U) = 2U\sqrt{p_x p_y} + p_x\alpha + p_y\beta$$

Use the Shephard Lemma to derive the compensated demand functions D_x^c and D_y^c :

$$\frac{\partial E(.)}{\partial p_x} = U\sqrt{\frac{p_y}{p_x}} + \alpha = D_x^c(p_x, p_y, U)$$

$$\frac{\partial E(.)}{\partial p_x} = U\sqrt{\frac{p_x}{p_y}} + \beta = D_y^c(p_x, p_y, U)$$

α and β are the minimal consumption amount that must be bought by the consumer.

Exercise H3 b)

Marshallian (uncompensated) demand function $D_i(p, E(p, \overline{U}))$

Hicksian (compensated) demand function $D_i^c(p, \overline{U}))$

$$D_i(p, E(p, \overline{U})) = D_i^c(p, \overline{U}))$$

Differentiate the above equation with respect to p_i :

$$\frac{\partial D_i}{\partial p_i} + \frac{\partial E(.)}{\partial p_i}\frac{\partial D_i}{\partial I} = \frac{\partial D_i^c}{\partial p_i}$$

By the Shephard Lemma, we know that $\frac{\partial E(.)}{\partial p_i} = x_i$, then:

$$\frac{\partial D_i}{\partial p_i} = \frac{\partial D_i^c}{\partial p_i} - x_i\frac{\partial D_i}{\partial I}$$

The first RHS term corresponds to the substitution effect while the second RHS to the income effect. The substituion effect is always negative because $\frac{\partial D_i^c}{\partial p_i}$ is the second derivative of the expenditure function which is concave. The income effect can be positive (normal good) or negative (inferior good).

Total Effect		Substitution Effect		Income Effect	Type of good
$\frac{\partial D_i}{\partial p_i}$	$=$	$\frac{\partial D_i^c}{\partial p_i}$	-	$x_i\frac{\partial D_i}{\partial I}$	
(-)		(-)	-	(+)	Normal
(-)		(-)	-	(-)	Inferior
(+)		(-)	-	(-)	Giffen

Exercise H4 1)

1. a.

$$Max\, U(x,y) = x_1^{\alpha}x_2^{\beta}x_3^{\gamma}$$

$$st\ :\ I = p_1x_1 + p_2x_2 + p_3x_3$$

Set the Lagrange

$$£ = x_1^{\alpha}x_2^{\beta}x_3^{\gamma} - \lambda(p_1x_1 + p_2x_2 + p_3x_3 - I)$$

and derive the uncompensated demand function D_1, D_2, D_3 :

$$D_1(p_1,p_2,p_3,I) = \frac{\alpha I}{p_1}$$

$$D_2(p_1,p_2,p_3,I) = \frac{\beta I}{p_3}$$

$$D_3(p_1,p_2,p_3,I) = \frac{\gamma I}{p_3}$$

Plug these unconpensated demands into the Utility function to get the Indirect Utility fonction V:

$$V(p_1,p_2,p_3,I) = \left(\frac{\alpha I}{p_1}\right)^{\alpha}\left(\frac{\beta I}{p_3}\right)^{\beta}\left(\frac{\gamma I}{p_3}\right)^{\gamma}$$

$$V(p_1,p_2,p_3,I) = I\left(\frac{\alpha}{p_1}\right)^{\alpha}\left(\frac{\beta}{p_3}\right)^{\beta}\left(\frac{\gamma}{p_3}\right)^{\gamma} \text{ with } \alpha+\beta+\gamma = 1$$

Reverse to obtain the Expenditure function E:

$$E(p_1,p_2,p_3,U) = \frac{U}{\left(\frac{\alpha}{p_1}\right)^{\alpha}\left(\frac{\beta}{p_3}\right)^{\beta}\left(\frac{\gamma}{p_3}\right)^{\gamma}}$$

b. $U(x_1,x_2) = \min(x_1,x_2)$. Perfect complement

$$I = p_1x_1 + p_2x_2$$

$$D_1(p_1,p_2,I) = \frac{I}{p_1+p_2}$$

$$D_2(p_1,p_2,I) = \frac{I}{p_1+p_2}$$

Plug these unconpensated demands into the Utility function to get the Indirect Utility fonction V:

$$V(p_1,p_2,I) = \frac{I}{p_1+p_2}$$

Reverse to obtain the Expenditure function E:

$$E(p_1,p_2,U) = U(p_1+p_2)$$

c. $U(x_1,x_2) = x_1+x_2$. Perfect subsitute, there are 3 cases:

$$I = p_1x_1 + p_2x_2$$

$$p_1 > p_2 \Rightarrow D_1(p_1,p_2,I) = 0; D_2(p_1,p_2,I) = \frac{I}{p_2}$$

$$p_1 = p_2 \Rightarrow D_1(p_1,p_2,I) = D_2(p_1,p_2,I) = \frac{I}{p_2} = \frac{I}{p_1}$$

$$p_1 < p_2 \Rightarrow D_1(p_1,p_2,I) = \frac{I}{p_1}; D_2(p_1,p_2,I) = 0$$