Handout 2: Hints

Exercise H2 2)

$$U_G(x,y) = \sqrt{x}\sqrt{y}$$

$$U_L(x,y) = \ln(x) + \ln(y)$$

Show that the MRS of both Utility function is the same and equal to:

$$MRS_G = MRS_L = \frac{y}{x}$$

Exercise H2 3)

$$\begin{cases} \underset{x,y}{Max} \;\; U(x,y) = \sqrt{x} + \sqrt{y} \\ s.\,t\; I = px + qy \end{cases}$$

1.

$$MRS = \left(\frac{y}{x}\right)^{0,5} = \frac{p}{q}$$

$$\Rightarrow y = \left(\frac{p}{q}\right)^2 x$$

Is constant along any ray from the origin

2. Compute first the Demand function

$$D_x(p,q,I) = \frac{I}{p(1+\frac{p}{q})}$$

$$D_y(p,q,I) = \frac{I}{q(1+\frac{q}{p})}$$

Expenditure share of x:

$$\frac{px}{I} = \frac{q}{q+p}$$

Expenditure share of y:

$$\frac{py}{I} = \frac{p}{p+q}$$

3. Income elasticity of x:

$$\frac{\partial x}{\partial I}\frac{I}{x} = 1$$

Income elasticity of x:

$$\frac{\partial y}{\partial I}\frac{I}{y} = 1$$

Exercise H2 4)

$$U(x_1,x_2) = x_1^{\alpha}x_2^{\beta}$$

$$\alpha + \beta = 1$$

1. Uncompensated Demand. Since $\alpha + \beta = 1$, you can derive directly

the uncompensated demand function:

$$D_{x_1}(p_1, p_2, I) = \frac{\alpha I}{p_1}$$

$$D_{x_2}(p_1, p_2, I) = \frac{\beta I}{p_2}$$

2. Compensated Demand.

$$\underset{x_1,x_2}{Min} \; I = p_1 x_1 + p_2 x_2$$

$$s.t. \; \overline{U} = x_1^{\alpha} x_2^{\beta}$$

Set the Lagrangian:

$$£ = p_1 x_1 + p_2 x_2 - \lambda\left(\overline{U} - x_1^{\alpha} x_2^{\beta}\right)$$

Derive wrt x_1, x_2 and λ.

$$\begin{cases} \frac{\partial £}{\partial x_1} = p_1 - \lambda \alpha x_1^{\alpha-1} x_2^{\beta} \overset{!}{=} 0 & (1) \\ \frac{\partial £}{\partial x_2} = p_2 - \lambda \beta x_1^{\alpha} x_2^{\beta-1} \overset{!}{=} 0 & (2) \\ \frac{\partial £}{\partial \lambda} = \overline{U} - x_1^{\alpha} x_2^{\beta} \overset{!}{=} 0 & (3) \end{cases}$$

From (1) and (2)

$$MRS = \frac{p_1}{p_2} = \frac{\alpha}{\beta} \frac{x_2}{x_1}$$

$$x_2 = \frac{\beta}{\alpha} \frac{p_1}{p_2} x_1$$

substituting into (3) give us the uncompensated demand functions:

$$D^c_{x_1}(p_1, p_2, U) = U\left(\frac{\alpha}{\beta}\right)^{\beta}\left(\frac{p_2}{p_1}\right)^{\beta}$$

$$D^c_{x_2}(p_1, p_2, U) = U\left(\frac{\beta}{\alpha}\right)^{\alpha}\left(\frac{p_1}{p_2}\right)^{\alpha}$$

3. Indirect Utility. Substitute the uncompensated demand function into the utility function

$$V(p_1, p_2, I) = I\left(\frac{\alpha}{p_1}\right)^{\alpha}\left(\frac{\beta}{p_2}\right)^{\beta}$$

4. Expenditure function. Reverse the Indirect Utility:

$$E(p_1, p_2, U) = U\left(\frac{p_1}{\alpha}\right)^{\alpha}\left(\frac{p_2}{\beta}\right)^{\beta}$$

5. Roy’s Identity: (don’t forget the sign!!!)

$$D_{x_1}(p_1,p_2,I) = -\frac{\frac{\partial V(.)}{\partial p_1}}{\frac{\partial V(.)}{\partial I}} = \frac{\alpha I}{p_1}$$

$$D_{x_2}(p_1,p_2,I) = -\frac{\frac{\partial V(.)}{\partial p_2}}{\frac{\partial V(.)}{\partial I}} = \frac{\beta I}{p_2}$$

6. Shephard lemma. Derive the expenditure function wrt prices to obtain the compensated demand function:

$$\frac{\partial E(.)}{\partial p_1} = U\alpha p_1^{\alpha-1}\alpha^{-\alpha}\left(\frac{p_2}{\beta}\right)^{\beta} = U\left(\frac{\alpha}{\beta}\right)^{\beta}\left(\frac{p_2}{p_1}\right)^{\beta} = D_{x_1}^{c}(p_1,p_2,U)\ with\,\alpha+\beta = 1$$

$$\frac{\partial E(.)}{\partial p_2} = U\left(\frac{\beta}{\alpha}\right)^{\alpha}\left(\frac{p_1}{p_2}\right)^{\alpha} = D_{x_2}^{c}(p_1,p_2,U)$$

7. Plug the indirect utility function in the compensated demand functions to get the uncompensated demand functions. Plug the expenditure function in the uncompensated demand functions to get the compensated demand functions.