Lecture 4: Aggregation and Partial Equilibrium

- Aggregation on the Production Side
- Aggregation of Consumers' demands
- Partial Equilibrium

Over the past few weeks we have analyzed single firms and then individual consumers. Now, we start putting things together. First we will investigate how to sum many firms' individual supply functions to obtain one aggregate supply curve. We will then repeat the same on the consumption side and ask under what conditions can individuals' demand functions be aggregated into one demand curve. Subsequently, given the aggregate demand and supply functions, we will focus on how demand and supply interact in one market. The resulting equilibrium is called a *partial* equilibrium because interactions with other markets are ignored (for the time being).

Aggregation on the Production Side

Recall a single firm's supply and factor demand functions resulting from its profit maximization:

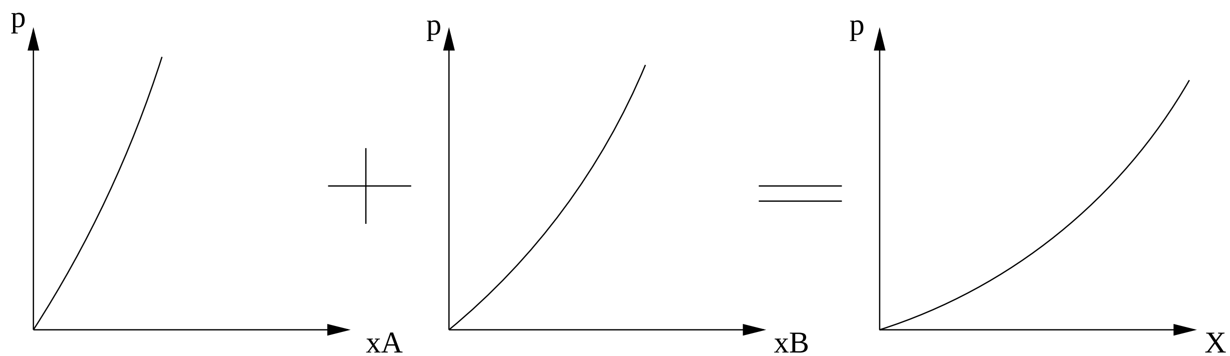
$$Y^i = Y^i(p, r, w, ...) \text{ and } K^i = K^i(p, r, w, ...), L^i = L^i(p, r, w, ...), ...$$

We will concentrate on aggregating the supply functions — the procedure is similar for factor demands.

Mathematically the aggregation is straightforward:

$$Y^{agg}(p, w, r, ...) = \sum_i Y^i(p, w, r, ...)$$

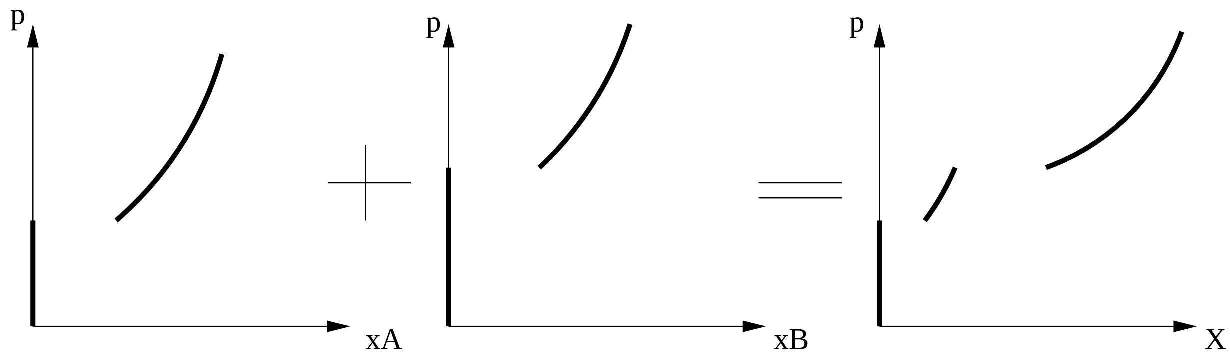
Graphically:



There are two problems worth pointing out, one quite fundamental and the other a small, potential, mathematical mishap:

- The standard graphical illustration of a production fct seems to involve IRS at low output and DRS thereafter. Note that such a function is not concave everywhere and therefore the mathematical solution resulting from optimization not necessarily a maximum. In other words, the second order sufficient conditions might be violated. The result is that below a certain (real) output price it is actually better to cease production and choose an output of zero.

Graphically:



As you see the jumps/discontinuities in the individual output supply functions give rise to discontinuities/holes in the aggregate supply curve. Now suppose the demand curve passes thru such a hole. So the fundamental problem that can arise here is non-existence of the equilibrium. This, of course, is due to non-convexities in production and not an inherent problem of aggregation. Quite to the contrary, aggregating non-convexities might render the aggregate production set convex again and thereby solve the problem.

- The second observation is just a helpful hint: Recall that the supply function — at least where it's continuous — is actually the marginal cost curve. Suppose instead of the individual supply functions you are given individual MC fcts and are asked to aggregate supply. But MC is expressed in terms of price as a function of quantity. So if you want to add these horizontally, you better invert them first, ie express quantity as a fct of price, and then add quantities. Do not add prices because then you would add vertically.

All told, the aggregation on the production side does not pose any particular problems as long as the production possiblities are convex (the prod fct concave). The same cannot be said of the consumption side to which we now turn.

Aggregation of Consumers' Demands

Recall the individual consumer's demand function for good j: $D_j^i(p_1, p_2, ..., I^i)$

We can again add them mathematically:

$$D_j^{agg}(p_1, p_2, ..., ???) = \sum_i D_j^i(p_1, p_2, ..., I^i)$$

The critical point here is that aggregate demand will in general not be a function of aggregate income ($I^{agg} = \sum_i I^i$) but instead of every individual's income I^i, ie the question marks above stand for $I^A + I^B + ... + I^Z + ...$. In other words, aggregate demand is in general not simply a fct of the price vector and aggregate income — as one might have suspected — but depends on the income distribution. Given the number of inhabitants in Germany, having 80,000,000+ arguments in there seems rather cumbersome. So how do we avoid it?

The honest answer to that question is: no way, sorry. The more pragmatic answer, however, involves the notion of homotheticity which we will briefly review/explain:

Homotheticity

Homotheticiy is a combination of two more familiar concepts: homogeneity and monotone transformations.

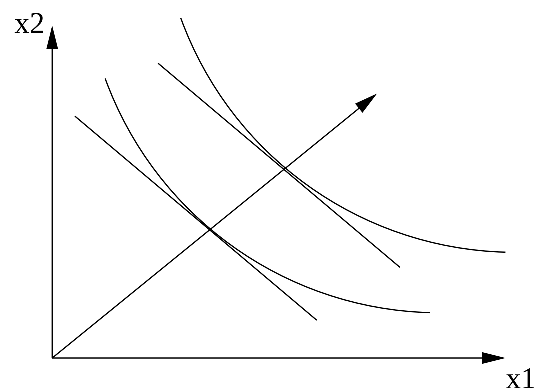
Def: a function is homothetic if it is a monotone transformation of a homogeneous function.

Note that the *if* above is actually an *iff* (= if and only if). Note also that a homogeneous function is automatically homothetic (since transforming the fct into itself is a monotone transformation) but not vice versa.

Now, where is the economics?

Let us discuss homogeneous utility fcts to begin with. Derivatives of homogeneous fcts are also homogeneous (although of one degree less) and ratios of homogeneous functions are also homogeneous with the degree of homogeneity equal to the difference of degrees of numerator and denominator. So a homogeneous utility function gives rise to a MRS which is homogeneous of degree zero (since numerator and denominator have the same degree). This means that such a MRS remains constant if you move northeast on any ray thru the origin. So if income increases (and prices remain unchanged) the consumption mix remains unchanged because the tangency point moves north-east on the ray and expenditure shares remain constant. Put differently, such a utility function implies income elasticities of one for every good.

Graphically:

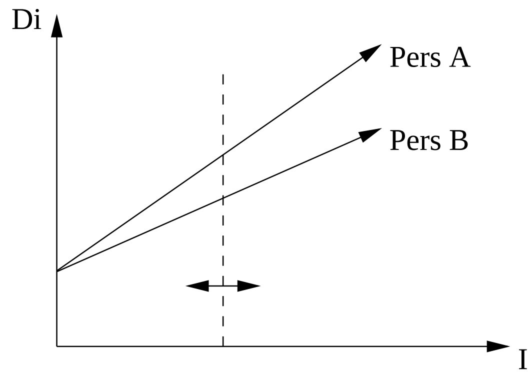


But the same is true for any homothetic utility fct. To see this recall that the same preferences can always be represented by a whole family of utility fcts which are related via monotone transformation. So if there is one member of the family that is homogeneous (and the definition of homotheticity makes sure there is) than every member of that family must possess the properties dicussed above because they all represent exactly the same preferences.

You may be familiar with Engel curves which graph demand as functions of income. Homotheticity implies that these curves are linear and pass thru the origin, ie are rays thru the origin. A slight generalization would be to allow vertical intercepts to differ from zero, ie have linear Engel curves that do not necessarily pass thru the origin. A utility function that gives rise to such Engel curves is called quasi-homothetic.

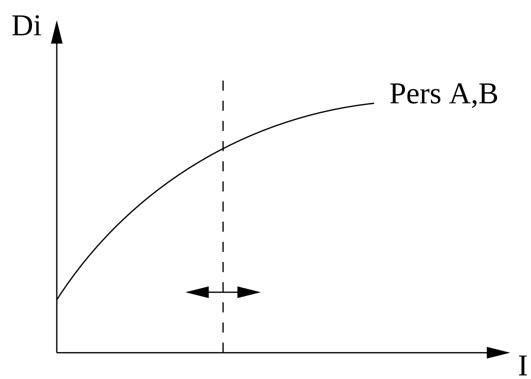
Now let us return to our goal of obtaining an aggregate demand fct that does only depend on aggregate income and not on its distribution.

Consider the following Engel curves:



Redistributing income between A and B affects demand (even though aggregate income stays unchanged) because the Engel curves (preferences) are not identical. So we need identical preferences.

Consider the following (identical) Engel curves:



It does not work either because they are not linear. So we also require the Engel curves to be linear, that is we require the utility fct to be quasi-homothetic.

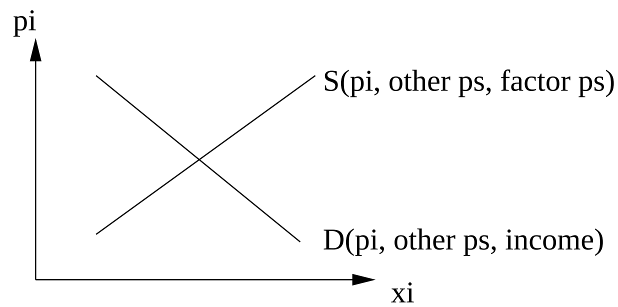
Only identical and quasi-homothetic preferences allow us to obtain aggregate demand fcts that depend on prices and aggregate income alone.

Are these assumptions realistic? Does it leave aggregate demand unchanged if you take away one billion from Mr. Gates and distribute it to all of us? Well, ask this question whenever someone presents you with aggregate demand fcts that do not depend on the income distribution.

Partial Equilibrium

We are now finally in a position to investigate how demand and supply interact. We start by studying this interaction in only one market. That is why this type of analysis is called partial equilibrium analysis. It is partial in the sense that we look only at one part/market of the whole market economy with its countless markets.

Obvioulsy, this is a simplification which makes the analysis easier and enables the researcher to obtain results that one would be unable to derive if one was considering all markets. On the other hand, this type of analysis ignores the numerous interactions between different markets. It depends on the good/market in question how important such interactions are, in other words how serious ignoring them turns out to be, and ultimately how appropriate partial equilibrium analysis of that market is.

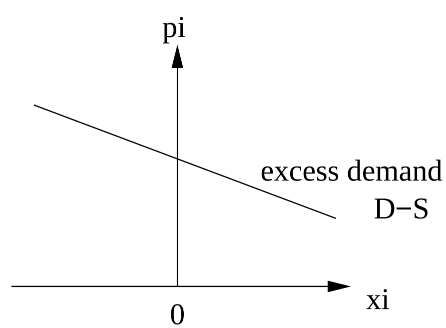


Consider the above diagram, sometimes called the economist's cross. Note that market or aggregate supply of good i as well as market/aggregate demand both depend on many variables. In the case of supply, these are the price of good i, factor prices, and possibly also prices of other goods if we go beyond our assumption of only one output good per production process. In the case of demand, these are the price of good i, the price of all other goods, aggregate income, plus possibly the entire income distribution (to avoid that nasty question).

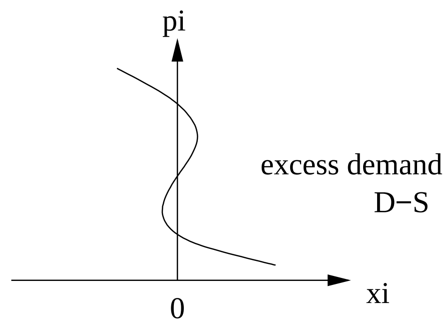
Despite so many arguments/variables, the above diagram depicts solely the dependence of demand and supply on the price of good i. Implicitely, all the other variables are assumed to be constant.

The market equilibrium is the intersection of demand and supply with the corresponding price and quantity. The location of this equilibrium depends on all the variables kept constant behind the scenes. An increase in income, for example, would shift demand east (if i is a normal good) and therefore affect the equilibrium.

One important question is whether the market — or rather the many isolated agents interacting anonymously in the market place — will find that equilibrium on their own. Technically, the question is whether the market equilibrium is stable. If it were not then we would be talking about a point that might never be reached. To answer the question, let us introduce the notion of excess demand. Excess demand is simply demand minus supply as a function of the own price.



The diagram depicts the excess demand curve corresponding to the previous diagram. Suppose, as seems reasonable, that price increases when there is excess demand and decreases when there is excess supply (= negative excess demand). Then we see that the above equilibrium is stable because as soon as the price moves away from its equilibrium value there is a tendency to return to that value.



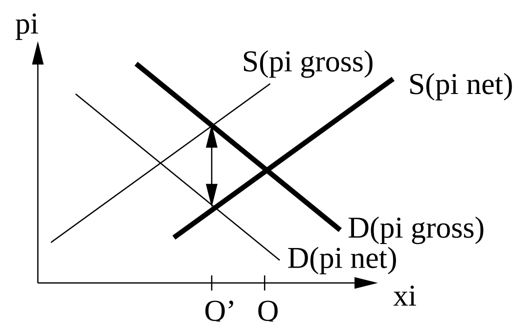
But the picture could look different — recall that theoretically there is the possibility of upwardsloping demand. There could be multiple equilibria and unstable ones.

In order for partial equilibrium analysis to make sense, excess demand should be downward sloping so that there exists only one stable equilbrium.

Applications of Partial Equilibrium

- **taxation**

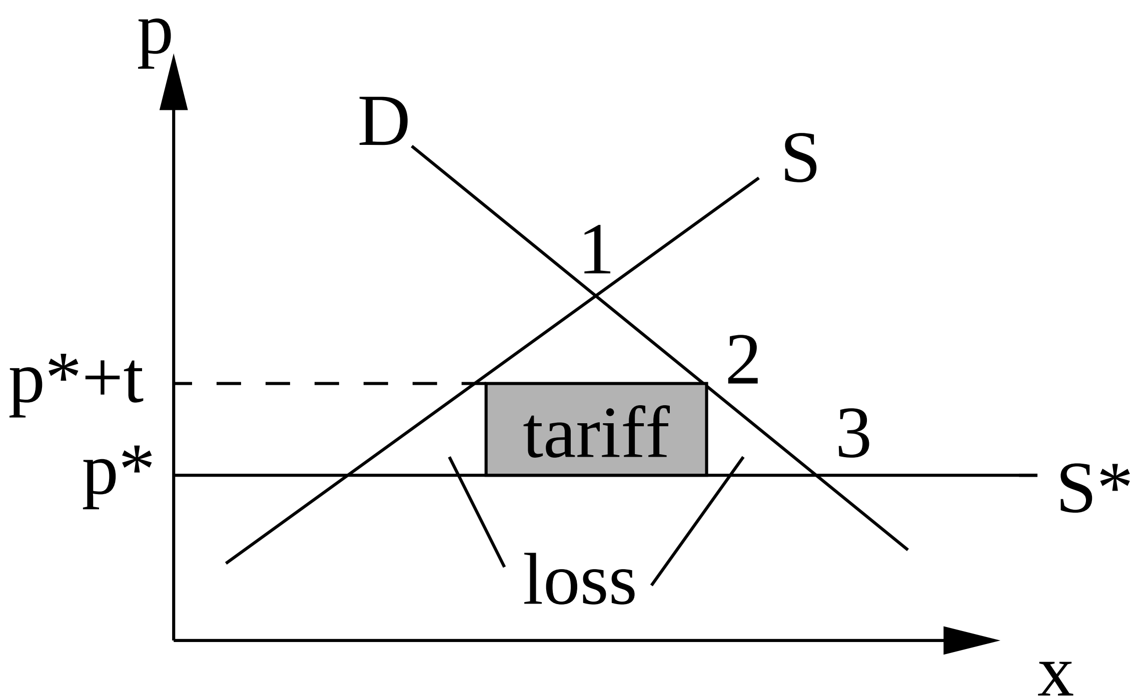
  Consider the market for a good on which a tax is levied. The tax could be an *ad varlorem* or value tax (ie a percentage of the price) or a quantity tax (ie 40 cents on a liter of gas) but in any case it drives a wedge between the price the seller receives and the price the buyer pays, forcing the former below the latter. So we need to distinguish two prices: the net price (net of tax) and the gross price (including the tax).



The above diagram contains the original demand and supply curves $D(p^{gross})$ and $S(p^{net})$. After the imposition of the tax, if you think in terms of the gross price p^{gross} then the supply curve shifts left. On the other hand, if we express everything in terms of the net price then demand shifts left. In any case the new equilibrium quantity is Q'. If you find shifting curves confusing then do not shift curves but instead take the tax wedge and push it between supply and demand coming from the left. If you do prefer to shift curves, make sure you never shift both of them to obtain an equilibrium to the left of Q' where both shifted curves intersect.

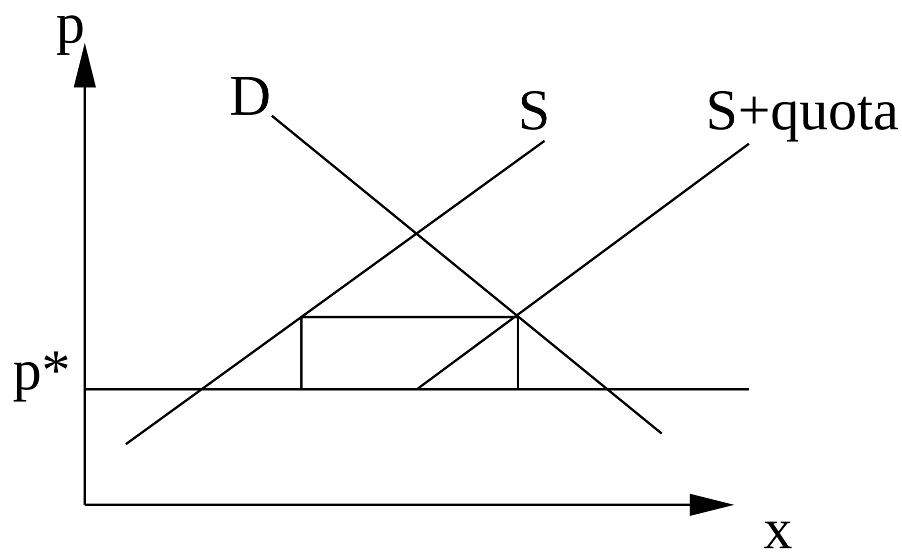
- **import tariff and VERs**

The most common simplification in the theory of international trade is the so-called small country assumption which is really an infinitessimal country assumption. The country is assumed to be so small as to not influence the world price level, not even passively. From the perspective of this infinitessimal country, we then have an infinitely elastic import supply at a fixed price coming to us from the rest of the world in addition to domestic supply.



The diagram shows the differences in price and quantity between the closed economy equilibrium (point 1), the free trade solution (point 3) and the tariff ridden equilibrium (point 2). Comparing free trade to the tariff equilibrium, note the import tariff revenue (the rectangle) and the efficieny loss (the two triangles). The efficiency loss is a genuine loss of welfare while the tariff revenue is simply a distributional effect, money taken away from consumers and ending up in the government's coffers.

Alternatively consider a quantity restriction on imports or a voluntary export restraint (VER):



The picture looks similar but the big question is who receives the rent resulting from the restriction (the rectangle that formerly constituted the tariff revenue). This quota rent could go to the local gov't if it auctions of the quotas, it could also end up abroad. In the case of a VER it definitely goes to the foreign country causing a further loss of domestic welfare on top of the efficiency loss.