# Lecture 3: Consumer Theory (cont'd)

- Price Effects — the Slutsky equation
- Testable Implications of the Model
- Welfare measurement
- Extension and Reinterpretation of the Model

We already investigated how changes in income affect demand. Uncompensated demands that is, because compensated demand functions do not depend on income. Now we want to investigate how price changes affect demand. Price changes affect uncompensated as well as compensated demand and we will derive a relationship between these two effects: the so-called Slutsky equation.

# Slutsky equation

Let us derive the Slutsky equation. As promised this is quite straightforward thanks to the dual approach. Start with:

$$D_i(p, E(p, \bar{U})) = D_i^c(p, \bar{U}) \qquad \forall i$$

Differentiate wrt $p_j$ to obtain:

$$\frac{\delta D_i}{\delta p_j} + \frac{\delta D_i}{\delta I}\frac{\delta E}{\delta p_j} = \frac{\delta D_i^c}{\delta p_j} \qquad \forall i, j$$

By Shephard's lemma $\delta E / \delta p_j = x_j$. Bringing the second term on the lefthand side (LHS) over to the right, ie subtracting the second term, gives the **Slutsky equation**:

$$\frac{\delta D_i}{\delta p_j} = \frac{\delta D_i^c}{\delta p_j} - x_j \frac{\delta D_i}{\delta I} \qquad \forall i, j$$

Note that we have derived the general form of the equation, in the sense that $i$ could equal $j$ or not. In case $i = j$ we speak of the *own-price* Slutsky equation. How does the change in its own price affect demand for a commodity. On the other hand, if $i \neq j$ we call this the *cross-price* Slutsky equation. How does the change in the price of another good affect demand for the commodity under consideration.

# Own Price Effect

Let us restate the own-price Slutsky equation:

$$\frac{\delta D_i}{\delta p_i} = \frac{\delta D_i^c}{\delta p_i} - x_i \frac{\delta D_i}{\delta I} \qquad \forall i$$

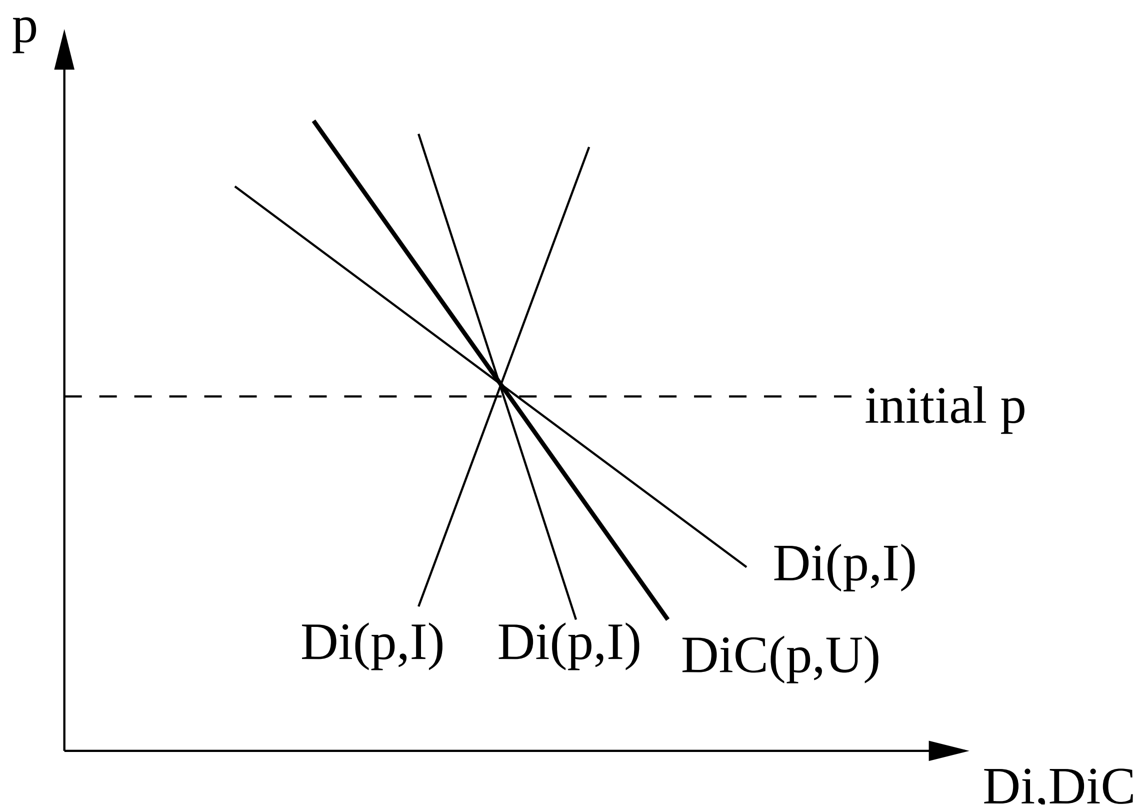
or, if you prefer elasticities:

$$\varepsilon_{D_i,p_i} = \varepsilon_{D_i^c,p_i} - \frac{x_i p_i}{I} \varepsilon_{D_i,I} \qquad \forall i$$

The own-price Slutsky equation tells us that the effect of a change in its own price on the (uncompensated) demand for a good can be decomposed into two effects: the two terms on the righthand side (RHS). That is why we sometimes speak of the Slutsky decomposition. The first term on the RHS is the substitution effect. Because the commodity is becoming more expensive, people substitute for it by buying other goods. Note that utility is kept constant here so that we can identify the pure effect. This effect is always negative which follows from the concavity of the expenditure function and the fact that $\delta D_i^c/\delta p_i$ is its second derivative. The second term on the RHS is the income effect. It is the implied change of income due to the price change. Since the first effect keeps utility constant — which cannot be because a price increase (decrease) would lower (increase) utility — we have an implied change in income left. The sign of this effect is ambiguous depending on whether it is an inferior or normal good. The following table summarizes the over-all effect of a price change on (uncompensated) demand:

| over-all effect | | substitution effect | | income effect | |
|---|---|---|---|---|---|
| $\boxed{-}$ | $=$ | $\boxed{-}$ | - | $\boxed{+}$ | normal good |
| $\boxed{-}$ | $=$ | $\boxed{-}$ | - | $\boxed{-}$ | inferior but not Giffen |
| $\boxed{+}$ | $=$ | $\boxed{-}$ | - | $\boxed{-}$ | Giffen good |

Graphically:



# Cross Price Effect

Let us restate the cross-price Slutsky equation:

$$\frac{\delta D_i}{\delta p_j} = \frac{\delta D_i^c}{\delta p_j} - x_j \frac{\delta D_i}{\delta I} \qquad \forall i \neq j$$

or in elasticities:

$$\varepsilon_{D_i,p_j} = \varepsilon_{D_i^c,p_j} - \frac{x_j p_j}{I} \varepsilon_{D_i,I} \qquad \forall i \neq j$$

The question here is whether the two goods $i$ and $j$ are substitutes or complements. Substitutes are goods like coffee and tea where one consumes either one or the other. The standard example for complements are nuts and bolts. One always needs/consumes both. Intuitively, a rise in the price of tea should increase the consumption of coffee, whereas a rise in the price of bolts also decreases the demand for nuts. In other words, if the cross-price effect is positive we speak of substitutes, if it is negative we speak of complements.

But we need to be more precise — do we mean uncompensated or compensated demand? That is, do we use $\delta D_i/\delta p_j$ or $\delta D_i^c/\delta p_j$ for our definition? If the effect on uncompensated demand is used, one speaks of gross substitutes/complements, while looking at the effect on compensated demand one speaks of net substitutes/complements. With the net definition, $\delta D_i/\delta p_j$ always exactly equals $\delta D_j/\delta p_i$ because they are both second derivatives of the expenditure function and it does not matter whether we differentiate first wrt $p_i$ and then wrt $p_j$ or the other way around. This is the symmetry property discussed below. With the gross definition on the other hand, $\delta D_i/\delta p_j \neq \delta D_j/\delta p_i$ in general due to the income effect. It can even be the case that both differ in sign which renders the gross definition completely useless. We therefore define substitutes/complements wrt the cross-price effect on compensated demands.

# Testable Implications of the Theory

- Adding up:

$$\frac{p_1 x_1}{I}\varepsilon_{x_1,I} + \frac{p_2 x_2}{I}\varepsilon_{x_2,I} + ... = 1$$

(derived from the budget constraint)

- Homogeneity:

$$\frac{\delta D_i}{\delta p_1}p_1 + \frac{\delta D_i}{\delta p_2}p_2 + ... + \frac{\delta D_i}{\delta I}I = 0$$

or

$$\varepsilon_{D_i,p_1} + \varepsilon_{D_i,p_2} + ... + \varepsilon_{D_i,I} = 0$$

(uncompensated demands are homogeneous of degree 0 and then apply Euler)

- Symmetry:

$$\begin{pmatrix} \frac{\delta D_1^c}{\delta p_1} & \frac{\delta D_2^c}{\delta p_1} \\ \frac{\delta D_2^c}{\delta p_1} & \frac{\delta D_2^c}{\delta p_2} \end{pmatrix} \quad \left( = \begin{pmatrix} E_{p_1 p_1} & E_{p_1 p_2} \\ E_{p_2 p_1} & E_{p_2 p_2} \end{pmatrix} \right)$$

is symmetric around the NW-SE diagonal

(it is the Hessian matrix or matrix of second derivatives of the expenditure fct which has this property because the order of differentiation does not matter)

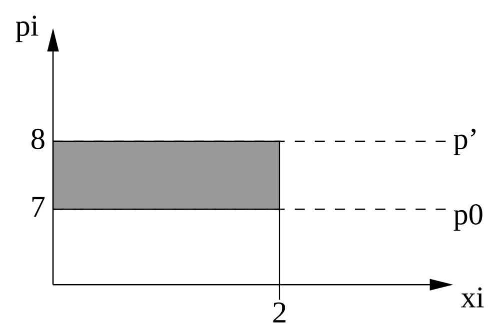
# Welfare Measurement

**Q:** what are the welfare implications of price changes?

This is a very important question in many areas, eg when the gov't changes (raises) taxes.

There are several answers to this question, listed below in increasing order of sophistication:

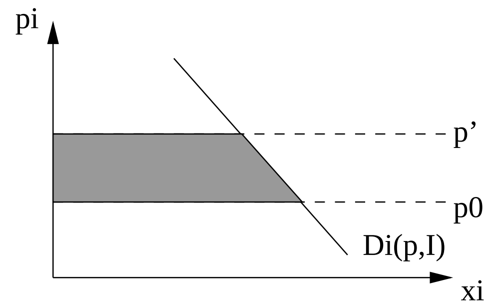
- naive answer: I go to the movies twice a month, the ticket price increases from 7 to 8 Euros so my welfare loss must equal 2 Euros, ie price change times quantity consumed (before/after the price change?).



- much better but not perfect: consumer surplus or the change in CS.

$$\triangle CS = \int_{\triangle p} D_i(p, I)dp$$

graphically (for one-dimensional price changes only):




Problems: CS itself might go to infinity — that's why we look at its change. More serious: moving along the Marshallian demand curve changes utility. That is, we measure the welfare effect of a price change keeping income constant but allowing utility to vary which does not make much sense.

- Variations — compensated and equivalent:

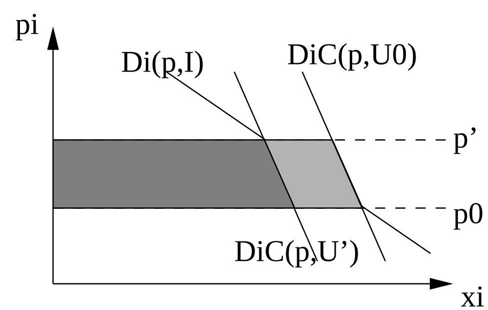
Compensated Variation:

$$CV = \int_{\triangle p} D_i^c(p, U^0)dp = E(p^1, U^0) - E(p^0, U^0)$$

Equivalent Variation:

$$EV = \int_{\triangle p} D_i^c(p, U^1)dp = E(p^1, U^1) - E(p^0, U^1)$$

and graphically:



# Extension and Reinterpretation of the Model

## buying and selling

Before we can talk about selling we need to talk about what is there to be sold. To this end, let us first explain the notion of endowments: so far the consumer received a fixed, exogeneous income $I$ (think of the check a student's parents send each month). Now, instead of a check he/she receives a box full of goodies, ie clothing, food — no beer, sorry. We call this the endowment point and denote it as $e = \{e_1, e_2, ...\}$. To attain his/her optimum our student will most likely sell some of the endowment and buy other goods (or more of other goods) instead, eg beer. That is, parents most likely do not know (or do not accept) his/her optimal consumption point and the student will therefore want to move along his budget constraint away from his/her endowment point by selling some endowments and buying things he/she prefers.

The budget constraint takes the form:

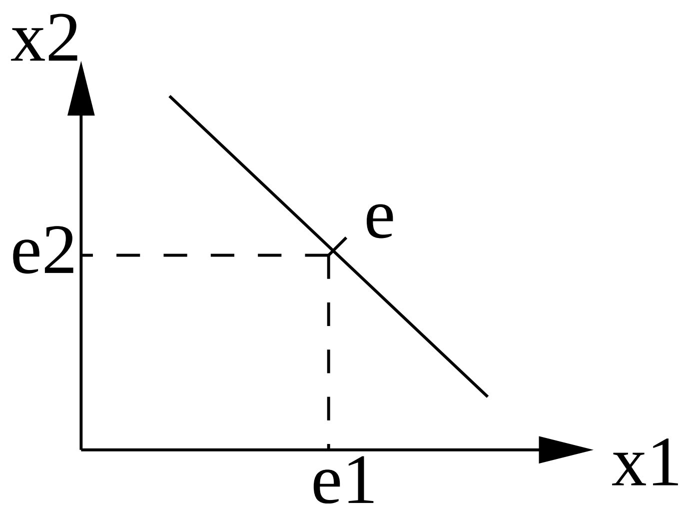
$$p_1 e_1 + p_2 e_2 = p_1 x_1 + p_2 x_2$$

or:

$$0 = p_1(x_1 - e_1) + p_2(x_2 - e_2)$$

where the expressions in parentheses are the net demands,
ie $x_i$ is the gross demand and $x_i - e_i$ the net demand.

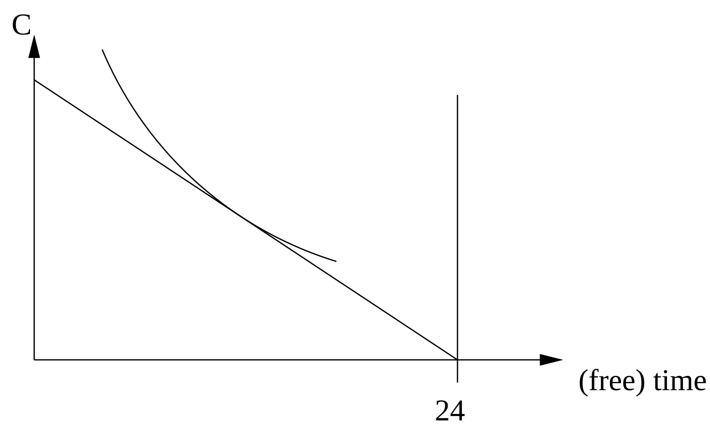
Graphically:




Note that a price change rotates the BC around the endowment point. It always has to pass thru that point because if you don't want to sell what your parents sent you nor buy other stuff then the price change does not affect you.

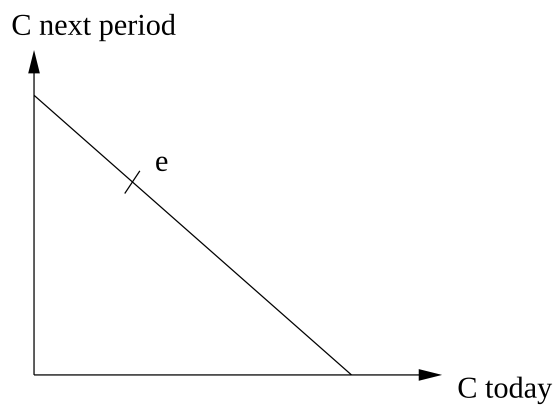
## labor supply

This is a simple reinterpretation of the model. The two commodities are time and consumption. The consumer is endowed with 24 hrs of time a day and can sell part of it (labor supply) while keeping the rest for sleeping/leisure. Prices are $p$, the price of consumption which one could normalize to one, and the price of time (or its opportunity cost) is the wage rate $w$. The relative price is then the real wage rate $w/p$.



## intertemporal

Here the two goods are consumption today and consumption next period. The endowment point is the vector of incomes (today and next period). For a student that endowment point will be located rather close to the consumption next period axis.



Pay attention to prices. $p_1$ is the price of consumption today and let us simply normalize it to one, ie $p_1 = 1$. $p_2$ is the price of consumption next period and the big question mark for the moment. Recall that the slope of the budget line is always the relative price of the two goods, ie $p_1/p_2$. What is the meaning of that relative price here? Well, it is still the ratio at which you can exchange good one for good two, only that now they are separated in time. How do you transfer your endowment of consumption today (ie income today) to next period? You take it to the bank and earn interest on it. That is you take an amount $s$ to the bank today and it will pay you back $(1+r)s$ next period. Note that $s$ could be negative in which case you take out a loan (implicite assumption being that the interest rate is the same for both, ie the bank earns no spread). So $p_1 = (1+r)p_2$ or $p_2 = 1/(1+r)$. In other words you discount future consumption in the budget constraint.