Mikro 1 — CAU Kiel
WS 2000/01 — Raff/Willmann

Midterm-Solution

1. Duality

a) $D_i^C = 0.5U\sqrt{p_j/p_i}$

b) $D_i = 0.5I/p_i$

c) There are two ways how to find the answer to b). For both you need the indirect utility function which can be obtained from the expenditure function by solving for U. Then we can either use Roy's identity to obtain the Marshallians or you take the Hicksians and plug in the indirect utility function.

d) $\delta D_i^C/\delta p_j = 0.25Up_i^{-1/2}p_j^{-1/2} > 0$, so they are net substitutes. In the two good scenario this has to be the case because we know that the Hicksians are homogeneous of degree zero in prices and therefore by Euler's formula their price derivatives weighted by the respective price sum to zero. Since the own price derivative is always negative (concavity of expenditure function) the (only) cross price derivative has to be positive.

e) $CV = 0.2U^0\sqrt{p_1p_2}$ and $EV = 0.2U^1\sqrt{p_1p_2}$

2. GE with production (Robinson Crusoe)

a) $L(p, w, I) = 0.5 - 0.5I/w$ and $C(p, w, I) = 0.5I/p + 0.5w/p$

b) $\pi = pC - wL = (p - w)L$ max! Careful b/c problem is linear. $C = L = 0$ if $w > p$, $C = L \in [0, \infty)$ if $w = p$, and $C = L \to \infty$ if $w < p$

c) $w/p = 1$, $\pi = I = 0$, $C = L = 0.5$

d) Consumer now faces price $1.1p$ and receives $I = 0.1pC$. Equilibrium is $w/p = 1$, $C = L = 10/21$ and this is not efficient b/c overall efficiency is violated and this new allocation is Pareto dominated by the old one from c).

3. Exchange Economy

a) Putting Heide in the southwest corner, Udo northeast, good 1 on the horizontal, and 2 on the vertical axis, the Edgeworth box is a 8×8 square, the endowment point is the southeast corner, and the contract curve looks like a one-step stair going up from Heide's corner half-way on the vertical axis, cutting horizontally across to the eastern edge and then going up there ending in Udo's corner.

b) $D_2^{H/U} = \sqrt{p_1/p_2}$. Equilibrium in the market for good 2 (and consequently due to Walras' law in both) if $2\sqrt{p_1/p_2} = 8$, ie $p_1/p_2 = 16$. Allocation is $x_2^H = x_2^U = 4$, $x_1^H = 7.75$, and $x_1^U = 1/4$.

c) Walras' law says that if (n-1) markets are in equilibrium then the nth market must also be in equlibrium. To verify, we know that Heide and Udo must stay within their budget constraints, that is $p_1x_1^{H/U} + p_2x_2^{H/U} = p_1e_1^{H/U} + p_2e_2^{H/U}$. Sum up these budget constraints, subtract the equilibrium condition for market 2 multiplied by p_2, ie $p_2x_2^H + p_2x_2^U = p_2e_2^H + p_2e_2^U$. and you are left — after dividing thru p_1 — with equilibrium in market 1. And vice versa.

d) It is possible to aggregate here because individual demands for good 2 do not depend on income (as we saw above, due to quasi-linearity), and all other income is spend on good 1, no matter whether by Heide or Udo. Aggregate demand: $x_1^{agg} = I^{agg}/p_1 - 2\sqrt{p_2/p_1}$ and $x_2^{agg} = 2\sqrt{p_1/p_2}$

4. Slutsky Equation

a) $D_i(p, E(p, U)) = D_i^C(p, U)$, differentiating with respect to p_j gives $\delta D_i/\delta p_j + (\delta D_i/\delta I)(\delta E/\delta p_j) = \delta D_i^C/\delta p_j$ and plugging in Shepard's lemma and subtracting the income effect results in the Slutsky equation: $\delta D_i/\delta p_j = \delta D_i^C/\delta p_j - x_j \delta D_i/\delta I$. (it's own price for $i = j$)

b) For a normal good the income derivative is positive, giving rise to a minus sign overall. If the good is inferior we have to distinguish whether the income effect overturns the substitution effect giving rise to a negative sign overall (Giffen good), or whether the substitution effect still dominates.

c) Now the quantity in the income effect is negative. If it's an inferior good, the overall effect is negative. If on the other hand it's normal then the income effect might dominate the substitution effect giving rise to a positive sign overall — or not (if it does not dominate).

d) By Shepard's lemma the Hicksians are the derivatives of the expenditure function. The derivatives of the Hicksians are therefore the second derivatives of the expenditure function and the order of differentiation does not matter.

e) Let's do it for good 1. $D_1 = \alpha I/p_1$ and the Hicksian $D_1^C = U(\alpha p_2/((1-\alpha)p_1))^{1-\alpha}$. Therefore the Slutsky equation takes the form $-\alpha I/p_1^2 = U((\alpha - 1)/p_1)(\alpha p_2/((1-\alpha)p_1))^{1-\alpha} - x_1\alpha/p_1$ from which we see on both sides that the own price effect in this case is negative. More specifically, it's the case of a normal good since we know that Cobb-Douglas is homothetic thus the income elasticity equals one.