Midterm - solution to 2 and 3 more explicit

Question 2: By now we know Robinson Crusoe's medical condition: he is schizophrenic and acts as a price-taking consumer/producer in a market economy despite being alone on his island. Production technology is given by the production function C = f(L) = L, where C stands for a consumption good and L for time worked, and his preferences can be represented by $U(C, L) = C^{1/2} (1-L)^{1/2}$. Note that time is measured in days and that he has an endowment of one unit of time.

a) Derive Robinson's (the consumer's) demand and labor supply functions.

Theory of the consumer. That is we formalize a person's preferences using a utility function, here $U(C, L) = C^{1/2} (1-L)^{1/2}$, and have him face a budget constraint which takes the form pC = wL + I. pC is consumption expenditure, wL actual labor income, and I some exogenous (for the consumer) monetary income, be it profits from a company he owns (we don't know yet that there won't be any profit) or some lump sum subsidy from part d). A few comments on the budget constraint: Note that we have written the budget constraint in monetary terms. One could as well normalize by setting p = 1 or dividing through by p and have the same equation in real terms, more precisely in units of C. Also, one could equivalently formulate it as pC + w(1-L) = w1 + I that is consider full income on the right hand side (w1 is what his endowment of one unit of time is worth) and then have "spending" on free time on the left hand side. Now, let's carry out the utility maximization which is simply a constraint optimization. The Lagrangean takes the form $\mathsf{L} = U(C, L) = C^{1/2} (1-L)^{1/2} + \lambda(pC - wL - I)$. The solution to this problem (you know how to do Cobb Douglas by now) is C(p, w, I) = 0.5 I/p + 0.5 w/p and L(p, w, I) = 0.5 - 0.5 I/w. That is his utility maximizing consumption demand and labor supply are functions of given prices and given exogenous monetary income.

b) Analyse the production behavior of Crusoe Inc. (be a bit careful here).

So let's be careful. Usually you would write down the definition of profit and maximize profit subject to the production function. You could do that by Lagrangean but it's always easier to plug in the production function (substitution) and in this case you get $\pi = pL - wL$. You immediately note that this is linear in L (could be written as (p-w)L after all) that is it's a line through the origin. Optimizing a straight line is tricky especially if it's not horizontal. Consider three cases: $p < w$, the company makes a loss on every unit it produces so chooses to produce nothing ($C = L = 0 = \pi$); p =w, now the line is horizontal and the company doesn't care how much they produce (still need to have C = L, of course) as profit is always zero ($C = L \in [0, \infty)$, $\pi = 0$); and thirdly $p > w$, that is positive profit on every unit so the company wants to produce more and more (C = L and π tend to ∞).

c) What will be the market outcome?

Usually you would set demand equal supply in n-1 markets (the nth market would be in equilibrium automagically) and get n-1 relative prices where in the simple case of two markets n-1 = 1. Here it's even easier or at least less mathematical. Consider the production side from b) with its three cases. The last one cannot be an equilibrium as the consumer could never supply an infinite amout of labor. So p cannot be greater than w in equilibrium. But the first case is no equilibrium either. To get the consumer to supply L = 0 and demand C = 0 when $I = \pi = 0$ is simply not possible no matter what's the relative price. That leaves p = w or w/p = 1 as the only potential equilibrium relative price. At this price and $I = \pi = 0$ the consumer demands/supplies C = 0.5 and L = 0.5 which is also optimal for the company, so we found the (only) market outcome.

d) Schizophrenia worsens. Self-proclaimed governor Crusoe introduces a 10 % consumption tax and rebates tax revenue lump-sum to consumer Robinson. What is the new market outcome? Is it efficient?

First, let's clarify notation. The consumption tax will drive a wedge between the price the producer receives and what the consumer pays. Let p denote the price for the producer (net of tax) so the consumer will face a price of 1.1p. The government (governor) collects the difference, ie 0.1pC and rebates the amount back to consumer Robinson, that is Robinson receives money income I = 0.1pC. Now, regarding the equilibrium, as under c) we can argue that the only potential equilibrium price is w/p = 1 (note that p is producer price). At this price the consumer demands C = 0.5 (0.1pC)/1.1p + 0.5 w/1.1p. Solving this for C gives C = 10/21. L = 0.5 - 0.5 (0.1pC)/w = 0.5 - (1/20).(10/21) = 10/21. This is also an optimum for the producer at this price so we have found the equilibrium.

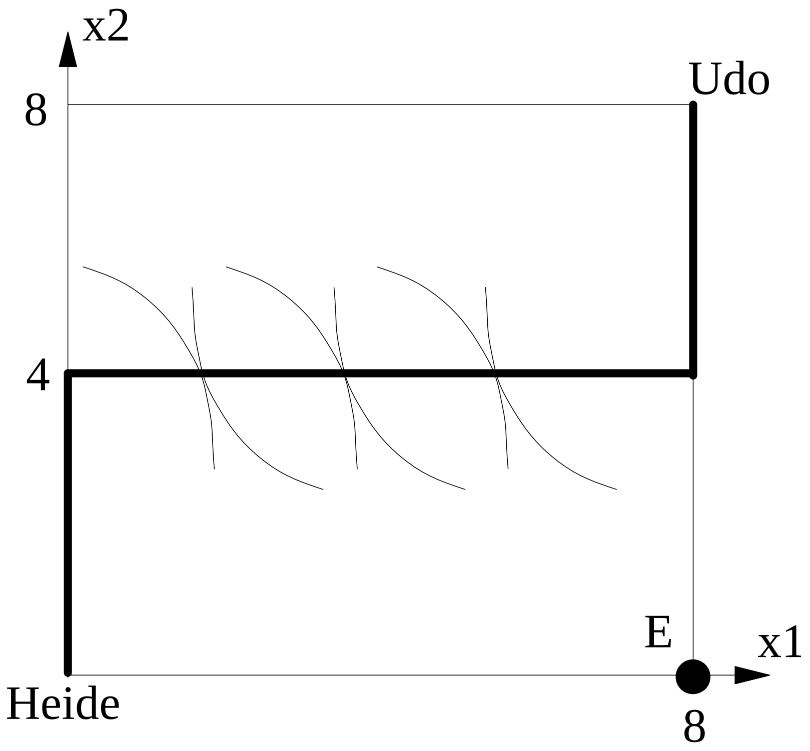
Question 3: Consider an exchange economy populated by Heide and Udo. Both of them have identical preferences which can be represented by $U(x_1, x_2) = x_1 - 1/x_2$, take prices as given, and have endowment vectors ($e_1^H=8$, $e_2^H=0$) and ($e_1^U=0$, $e_2^U=8$) respectively.

a) Draw the corresponding Edgeworth box (don't forget to label axes and Heide and Udo's corners), indicate the endowment point, and depict the specific (not just a generic) contract curve.

Seems that the challenge here was to find the contract curve (which, by the way, wasn't needed at all to answer the rest of this question). Obviously you were not expected to plot lots of indifference curves given the time constraint. But this utility function should have looked familiar. We discussed it in section. It's quasi-linear and once you derive the MRS ($= x_2^2$) you note that the slope of the indifference curves (that's what the MRS is) does not vary with x_1. So if we put x_1 on the horizontal axis the indifference curves will have the same slope along any horizontal line. Now, at which horizontal line are Heide's and Udo's indifference curves tangent? Because of symmetry it must be the middle one at $x_2 = 4$. Below (above) it both would prefer to trade northwest (southeast) so the whole thing looks as in the diagram on the following page.

b) Calculate the market outcome, ie equilibrium price and quantities.

We already saw that MRS = x_2^2 which must equal the relative price p_1/p_2 when the consumer maximizes utility (but feel free to go through the maximization). Since the MRS exceptionally does not depend on x_1 we can directly obtain the individual demand functions for good 2: $D_2^{H/U} = (p_1/p_2)^{1/2}$. Now, in equilibrium Heide's and Udo's gross demand for good 2 must equal what's out there, that is 8 units. So $2(p_1/p_2)^{1/2} = 8$ and solving for the relative price gives $p_1/p_2 = 16$. Of course, one could have obtained the same relative price by setting demand = supply/endowment in market one. Given this relative price, one can compute the equilibrium allocation (ie quantities) from the demand functions: $D_2 = 4$ for both and $D_1 = e_1 + (p_2/p_1)\, e_2 - (p_2/p_1)^{1/2}$ so $D_1 = 0.25$ for Udo and $D_1 = 7.75$ for Heide (the world just isn't fair).



c) State Walras' law and verify it for this economy.

Walras' law: when (n-1) markets are in equilibrium then the nth market must also be in equilibrium. To verify consider the following matrix:

$p_1\,(x_1^H - e_1^H)$	$p_2\,(x_2^H - e_2^H)$
$p_1\,(x_1^U - e_1^U)$	$p_2\,(x_2^U - e_2^U)$

Heide's and Udo's respective budget constraints imply that row one and two each sum to zero. The whole matrix therefore sums to zero. But then if either column sums to zero, the other column must do so, too. And a column summing to zero means equilibrium in the corresponding market. Simply divide away the price and you see more clearly that a column summing to zero means quantity demanded equal total endowment in the economy.

d) Abstracting from the endowments and assuming monetary income instead, can you aggregate their individual demands so that aggregate demand will be a function of aggregate income and will not depend on the income distribution? Justify your answer and demonstrate if possible.

One sees (or can check) that the utility function is not homogeneous. It isn't even homothetic because then the MRS would be constant along any ray through the origin, ie should be a function of x_1/x_2 which it is not. That leaves the possibility that it is quasi-homothetic - linear Engle curves not going through the origin. This is indeed the case as we can see from the demand curves derived under b). Demand for good two does not even depend on income so income distribution is definitely not an issue. Indeed we already saw under c) that aggregate demand for good two does not depend on income (aggregate or individual) at all: $D_2^{agg} = 2(p_1/p_2)^{1/2}$. Regarding the first good, $D_1 = I/p_1 - (p_2/p_1)^{1/2}$ for each individual which is clearly linear in I. And $D_1^{agg} = I^{agg}/p_1 - 2(p_2/p_1)^{1/2}$. So yes, aggregation is possible here (because of quasi-homotheticity).