Exercise Session 3

1 Problem: Walras' law

Suppose we have n markets. Prove that if there is market clearance in n-1 markets, then supply equals demand in the n^{th} market in the case of an exchange economy with Cobb-Douglas utility functions (Exercise 15.B.1).

2 Problem 16.E.1

Given a utility possibility set U, denote by $U' \subset U$ the subset of actually achieved by feasible allocations:

$$U' = \left\{ (u_1(x_1), \ldots, u_I(x_I)) : \sum_j y_j + \overline{\omega} \text{ for some } y_j \in Y_j \right\}$$

(Relative to U', the set U allows for free disposal of utility).

a) Give a two-consumer, two commodity exchange example showing that it is possible for a point U' to belong to the boundaty of U and *not* be a Pareto optimum.

b) Suppose that every Y_j satisfies free disposal and $0 \in Y_j$. Also, assume that for every i, $X_i = \mathbb{R}^L_+$ and $\succsim_i$ is continuous and strongly monotone. Show that any boundary point of U that belongs to U' is then a Pareto optimum. [*Hint*: Let $u_i(0) = 0$ for all i and show first that $U' = U \cap \mathbb{R}^L_+$. Next argue that if $u \in U$ is a Pareto optimum and $0 \leq u' \leq u$, $u' \neq u$, then we must be able to reach u' with a surplus of goods relative to u.]

c) Consider an exchange economy with consumption sets equal to $\mathbb{R}^L_+$, continuous, locally nonsatiated preferences, and a strictly positive endowment vector. Show that if $u = (u_1, \ldots, u_I)$ is the utility vector corresponding to a price quasiequilibrium with transfers then u cannot be in the interior of U; that is, there is no feasible allocation yielding higher utility to *every* consumer. [*Hint*: Show that $w_i > 0$ for some i and then apply Proposition 16.D.2.]