Homework # 2
### due May 14

Problem 1: Consider the framework used in section 16.F of MWG.

a) Explicitly derive the three types of conditions (16.F.4–16.F.6) for Pareto Optimality.

b) Choose appropriate prices and show that the respective optimality conditions of consumers and producers (16.F.10 and 16.F.11) correspond to the conditions you derived under a).

c) Standard definitions of price equilibria (e.g. Def 16.B.3) not only require consumption and production optimality, but also market clearing. Are conditions 16.F.10 and 16.F.11 sufficient for market clearing?

Problem 2: Consider a pure exchange economy consisting of two individuals, A and B, and two goods, x and y. A's preferences can be represented by the utility function $U_A(x, y) = xy$. B's preferences can be represented by the family of indifference curves that have the properties depicted in the following figure (there is one such curve for each k).
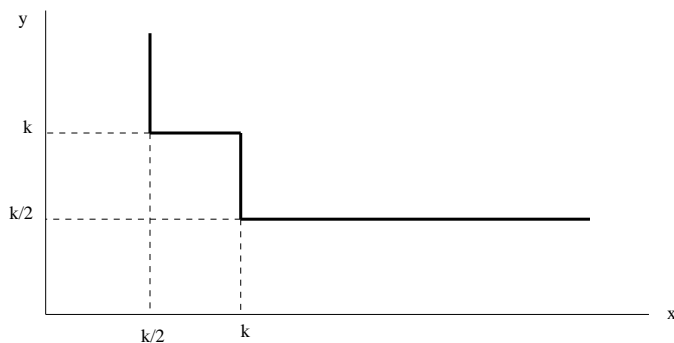

A is endowed with one unit of x, and no units of y. B is endowed with one unit of y, and no units of x.

a) Is there a competitive equilibrium for this economy? If yes, describe it (i.e. give prices and allocations). If no, explain why, and indicate which assumption used to establish existence is violated.

b) Suppose we "replicate" this economy eight times (that is, consider a new economy with 8 type A's, and 8 type B's). Is there a competitive equilibrium? If yes, describe it (give prices and allocations).

c) In this example, what is the smallest number of replications for which a competitive equilibrium exists?

Problem 3: Consider an economy with two goods, 1 and 2. Normalize prices so that $p_1 + p_2 = 1$. Suppose that excess demand functions $\xi_i(p_i)$ are continuous.

a) Suppose the aggregate excess demand for good 1 has the properties depicted in the following figure:
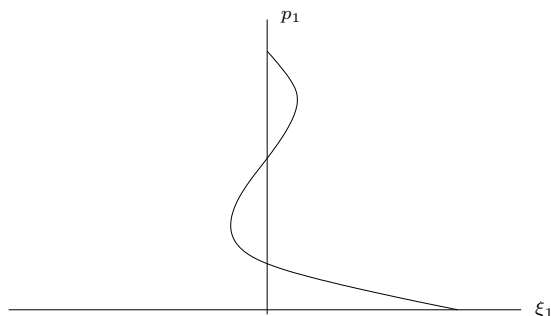
Sketch ξ_2 as a function of p_1 and indicate which prices are equilibria.

b) Suppose that the aggregate excess demand for good 1 has the proporties depicted in the next figure.
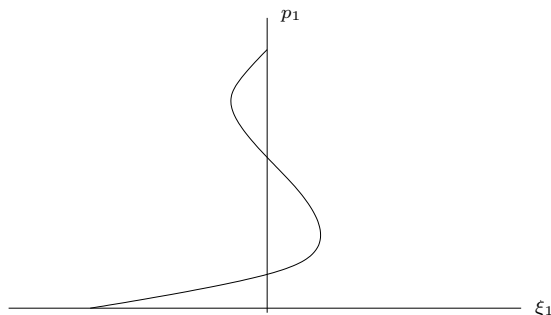
Again sketch ξ_2 and indicate equilibrium prices.

c) For this simple economy, prove, using only the intermediate value theorem (do not use a fixed point theorem) that equilibrium exists. (Hint: think about your pictures, what happens if $\xi(0) \leq 0$?)