KU Leuven
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Homework # 1

due Oct 29

Problem 1: Consider the utility fct $u(x, y, z) = \alpha \ln(x - a) + \beta \ln(y - b) + \gamma \ln(z - c)$, where $\alpha + \beta + \gamma = 1$.

a) Find the uncompensated demand functions and the indirect utility function. Verify that they satisfy the properties given in propositions 3.D.2. and 3.D.3., and provide an economic interpretation of the parameters a, b, and c.

b) Find the compensated demand functions and the expenditure function. Verify that they satisfy the properties given in propositions 3.E.2 and 3.E.3.

c) Show that the price derivatives of the expenditure function are the Hicksian demands, that the own-price derivatives of the Hicksians are negative, that the cross-price derivatives of the Hicksians are symmetric, and that the matrix of substitution effects is negative semi-definite.

Problem 2: A freshly minted KUL graduate has the utility function $U = \ln x_t + \beta \ln x_{t+1}$, where x_t is the consumption during her studies and x_{t+1} the consumption after graduation. Similarly, denote by e_t the income during her studies, and by e_{t+1} her income after graduation. Finally, let r be the interest rate between periods t and $t + 1$.

a) Write down the budget constraint. Suppose that without studying she could have earned $(e_t = 200,000, e_{t+1} = 300,000)$. While studying, her income was only $e_t = 60,000$. The interest rate is 25 percent. How much does she have to earn after graduation to make her studies worthwhile.

b) Derive her uncompensated demands. Assume the same incomes and interest rate as in a) and suppose $\beta = 1.2$. Does she save or borrow during her studies?

c) The government offers her a subsidized loan at an interest rate of 10 percent. How high is the equivalent variation of this subsidy, i.e. what income subsidy during her studies would lead to the same additional utility.

Problem 3: You know that an individual's expenditure function is given by $E(p_1, p_2, U) = U p_1^{\alpha} p_2^{\beta}$.

a) What restrictions (if any) on the parameters α and β are required to ensure that $E(.)$ constitutes a valid expenditure function. Assume that the restriction(s) are/is satisfied.

b) Find the indirect utility function, the Hicksian demand functions, and the uncompensated demands. Carefully list the correct arguments for each function.

c) Use an alternative approach — i.e. different to what you did in b) — to calculate the uncompensated demand functions.

d) Are goods 1 and 2 complements or substitutes? Could it be otherwise? Please justify your answers.