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Exercises # 2

Problem 1: A consumer has utility function

$$u(c,x) = \min\{c,x\},$$

where c is consumption and x is leisure. The consumer has T units of time and receives wage w for each unit of time she works. Wages are the only source of income. Use L to denote the number of units of time spent working, so that $x = T - L$. The price of consumption is p.

a) Find the Marshallian demands for consumption and leisure.

b) Find the indirect utility function and the expenditure function.

c) Draw a graph of the Marshallian labor supply curve. For our purposes here, let's say that labor supply is *backward bending* if there is a point at which an increase in w leads to a decline in L. Does this occur here?

d) Suppose that Parliament proposes to impose a tax on labor income and throw the proceeds in the North Sea. Explain whether this will encourage or discourage time spent working (in the context of this model).

Problem 2: MWG 3.G.4, do the whole problem, but with the following edits:

a) The wording in MWG is a bit unclear. I want you to show the following. Suppose that u is additively separable and $g(u)$ is a monotonic, differentiable affine function. (That is, $g(u) = au + b$ for constants and b.) Prove that the transformed utility $g(u(x))$ has an additively separable representation:

$$g(u(x)) = \sum_{i=1}^{N} g_i(x_i)$$

(The following converse is true, but you don't have to prove it: if g is monotonic and differentiable, then $g(u(x))$ additively separable implies g affine.)

b) Write a consumption bundle $\mathbf{x}$ over all N goods as $\left(\mathbf{x}^k, \mathbf{x}^{N-k}\right)$, where $\mathbf{x}^k = (x_1, x_2, ..., x_k)$ is the vector of the first k goods, and $\mathbf{x}^{N-1} = (x_{k+1}, ..., x_N)$ is the vector of the remaining $N-k$ goods. Consider arbitrary consumption bundles (a,c), (a,d), (b,c), and (b,d). Prove that $(a,c) \succsim (b,c)$ if and only if $(a,d) \succsim (b,d)$. (That is, if we compare two bundles that differ only on the first k goods, our preference between them doesn't depend on whether they both give us c or they both give us d for the final $N-k$ goods.)

c) Don't worry about the Walrasian/Hicksian distinction – just show that there can be no inferior goods.

d) You need to keep assuming $u_l(\cdot)$ strictly concave here.

Problem 3: Let $U(x, y) = x - 1/y$. Derive the cross price elasticity of both (uncompensated) demands. Can you say whether they are substitutes or complments?