D0C08A/D0N27a/D0C29a — KU Leuven
Fall 2009 — Gerald Willmann

# practise final

(actual exam given at another school where students had 2 hr 45 min)

**Problem 1:** Lizzie is a utility maximizer. She consumes 4 goods: apples (A), books (B), crackers (C), and diapers (D). For each apple consumed, she consumes exactly one book. For each cracker consumed, she uses exactly one diaper. *For any prices and income* Lizzie spends exactly half of her (money) allowance (M) on apples and books and half on crackers and diapers.

a) Define the class of utility functions consistent with Lizzie's behavior.

b) Solve for the demand functions for apples and for crackers.

c) Sketch a price offer curve for good A in (A,B) space. Label carefully.

**Problem 2:** State whether each of the following is true or false. If true, provide a proof; if false, provide a counter-example or a clear explanation.

a) A strictly monotonic Bernoulli utility function, $u(w)$, is strictly concave in $w$ if and only if $r(w) > 0$ (where $r(\cdot)$ is the Arrow-Pratt measure of absolute risk aversion).

b) $V(p_1, p_2, m) = p_1^{\alpha} p_2^{1-\alpha} m$ is a valid indirect utility function.

c) If a twice continuously differentiable utility function, $u(x, y)$, is h.d.1 in $x$ and $y$, then preferences cannot be strictly convex.

**Problem 3:** You have been appointed by the chair of the economics department to evaluate whether your macro professor has gone completely bananas. (Rumors suggest a recent cohort of graduate students may have pushed him beyond his transversality condition.) The chair has established that your professor has convex, strongly monotone preferences, but knows nothing more. The professor in question consumes just 2 goods, tropical fruit and beer, which have prices $p_f$ and $p_b$, respectively. The chair has observed the following data over the past four years. For convenience, the price of fruit has been normalized to one.

| Year | $(p_B, p_F)$ | $(x_B, x_F)$ |
|---|---|---|
| 2009 | (10,1) | (2,80) |
| 2008 | (8,1) | (10,16) |
| 2007 | (3,1) | (20,2) |
| 2006 | ($\frac{1}{3}$,1) | (6,8) |

Determine whether the data satisfy WARP, SARP, and/or GARP. *For each axiom* you must identify *all* violations (if any). Be careful, concise, and complete.

**Problem 4:** Let $U(s,n) = \sqrt{s+2n}$ represent Abby's preferences, where $s$ denotes sardines eaten per day and $n$ represents the number of daily naps. Abby currently lives on Rugby Avenue, where she has a daily allowance of 1000 woozels (woozels are the unit of currency in her household). Prices in her Rugby Avenue home are $q_s = 10$ and $q_n = 20$ for sardines and naps, respectively. Abby has been wandering the neighborhood and now has an offer to move to a home on Westwood Avenue where her allowance would be 2000 woozles per day, and prices $p_s = 10$ and $p_n = 50$, respectively.

a) Find Abby's indirect utility function, $v(\vec{p}, w)$ and expenditure function $e(\vec{p}, u)$.

b) Calculate Abby's indirect utilty function $\mu(\vec{p}; \vec{q}, w)$.

c) Would Abby be willing to move to Westwood Avenue? If not, how high would Abby's woozel allowance have to be in her new home for her to be willing to move? Or if she is willing to move, then how much would her current family have to increase her woozel allowance to convince her to stay? Note whether your answer is compensating or equivalent variation.

d) What, if anything, can you say about Abby's optimal consumption pattern in her current home relative to her potential new home? Be as specific as you can.

**Problem 5:** Santa's elves produce toys for good little girls and boys each year, but the toy quality can vary. The elves care only about the children's utility, which is given by the the (Bernoulli) utility function $U(q) = \sqrt{q}$, where $q$ denotes quality.

Quality is a random variable with the following distribution:

$$q = \begin{cases} 0, & \text{with 30\% probability;} \\ 25, & \text{with 20\% probability;} \\ 64, & \text{with 25\% probability, and} \\ 100, & \text{with 25\% probability.} \end{cases}$$

Assume that all toys take the same quality in a given year, and normalize the mass of good little girls and boys to one.

a) Find the elves' expected utility of quality, and compare this to the expected value of quality. Sketch the lottery on a graph with quality on the horizontal axis. Label your graph completely.

b) Finneas D. Grinch, LLP (FDG) is the management arm of Santa's north pole operation center. FDG offers to install its patented "Toy-tester" service, which will screen out the worst quality toys with probability one. Unfortunately, the Toy-tester also interferes with the elves' regular work, which makes it impossible to produce the highest quality toys, and changes the probability of producing the two intermediate qualities as follows:

$$q = \begin{cases} 25, & \text{with probability } p > 0 \\ 64, & \text{otherwise.} \end{cases}$$

What restrictions (if any) must $p$ satisfy in order for the elves to be willing to use the Toy-Tester technology? Calculate your answer AND identify it on a carefully labeled graph. (You must show your work for full credit, and you must label all relevant points on the graph.)

c) Suppose that the Toy-tester is designed such that $p = \frac{1}{2}$, but it also imposes a fixed "quality cost", $c > 0$ from running the toys through the testing device. (The Toy-tester scratches the paint on the underside of each toy; children notice these things.) What is the maximum value that $c$ can take so that the elves are still willing to use the Toy-tester? (*Hint: You may leave your answer in the implicit form – i.e. no need to solve for $c$ explicitly as long as you set up the problem correctly.*) **Draw a new graph and label $c$ carefully.**

d) Finally, now suppose instead that toy quality is independently drawn from the distribution for each child. What, if anything, can you say about the elves' decision to use the Toy-tester technology as the number of good little children increases (holding the population mass fixed at one)? Be as precise as you can.