## Exercises # 2

## **Problem 1:** A consumer has utility function

$$u(c, x) = \min\{c, x\},\$$

where c is consumption and x is leisure. The consumer has T units of time and receives wage w for each unit of time she works. Wages are the only source of income. Use L to denote the number of units of time spent working, so that x = T - L. The price of consumption is p.

- a) Find the Marshallian demands for consumption and leisure.
- b) Find the indirect utility function and the expenditure function.
- c) Draw a graph of the Marshallian labor supply curve. For our purposes here, let's say that labor supply is *backward bending* if there is a point at which an increase in w leads to a decline in L. Does this occur here?
- d) Suppose that Parliament proposes to impose a tax on labor income and throw the proceeds in the North Sea. Explain whether this will encourage or discourage time spent working (in the context of this model).

## **Problem 2:** MWG 3.G.4, do the whole problem, but with the following edits:

a) The wording in MWG is a bit unclear. I want you to show the following. Suppose that u is additively separable and g(u) is a monotonic, differentiable affine function. (That is, g(u) = au + b for constants and b.) Prove that the transformed utility g(u(x)) has an additively separable representation:

$$g(u(x)) = \sum_{i=1}^{N} g_i(x_i)$$

(The following converse is true, but you don't have to prove it: if g is monotonic and differentiable, then  $g\left(u\left(x\right)\right)$  additively separable implies g affine.)

b) Write a consumption bundle  $\mathbf{x}$  over all N goods as  $(\mathbf{x}^k, \mathbf{x}^{N-k})$ , where  $\mathbf{x}^k = (x_1, x_2, ..., x_k)$  is the vector of the first k goods, and  $\mathbf{x}^{N-1} = (x_{k+1}, ..., x_N)$  is the vector of the remaining N-k goods. Consider arbitrary consumption bundles (a,c), (a,d), (b,c), and (b,d). Prove that  $(a,c) \succsim (b,c)$  if and only if  $(a,d) \succsim (b,d)$ . (That is, if we compare two bundles that differ only on the first k goods, our preference between them doesn't depend on whether they both give us c or they both give us d for the final N-k goods.)

- c) Don't worry about the Walrasian/Hicksian distinction just show that there can be no inferior goods.
- d) You need to keep assuming  $u_l\left(\cdot\right)$  strictly concave here.

**Problem 3:** Let U(x,y) = x - 1/y. Derive the cross price elasticity of both (uncompensated) demands. Can you say whether they are substitutes or complements?