Homework 2   due 8/9 in class

1. Suppose a firm is producing a single output y using a single input k and takes input and output price as given. Its production technology exhibits increasing returns to scale for low output levels but decreasing returns past some quantity of production.

   a) Graph its production function.

   b) Illustrate how one can graphically derive its profit-maximizing output supply curve from this production function.

   c) Illustrate how one can graphically derive the cost function from the production function.

   d) Show how one can derive the marginal and average cost functions from your answer to c).

   e) Graphically derive the supply curve from your answer to d).

   f) Are the two supply curves derived above identical (or should they be identical :) ? Why or why not?

2. Consider a firm with the Cobb-Douglas production function $Y = F(K, L) = K^\alpha L^\beta$. Let r and w denote the prices of the two input factors and p the price of the firm's output.

   a) Find the conditions on the two parameters α and β ensuring that

   i) the marginal products MP_K and MP_L are diminishing;

   ii) there are constant/increasing/decreasing returns to scale.

   b) For $\alpha + \beta = 1$ find the firm's minimum cost C(r,w,Y) and cost-minimizing input demands K(r,w,Y) and L(r,w,Y).

   Now assume $\alpha + \beta < 1$.

   c) Describe in minute detail one way of obtaining the profit maximizing output supply and the unconditional input demands as functions of r, w, and p.

   d) Do you know of another way - also describe in minute detail.

   e) Discuss the profit maximizing output supply when $\alpha + \beta \geq 1$.

3. Consider a price taking, profit maximizing firm with a production technology as described in the first question. Analyze how its output supply is affected by

   a) a lump-sum tax of a fixed amount which is payable if the firm continues in business;

   b) a proportional tax on profits;

   c) a tax on each unit of output.

4. The domestic aggregate demand and supply curves for a commodity are given by $D = \alpha - \beta P$ and $S = \gamma P$ respectively, where α, β, and γ are positive constants.

a) Find the domestic market equilibrium (price and quantity) if neither imports nor exports are allowed.

b) Suppose there is free trade in this commodity at a fixed world price of Pw. Comparing Pw to the domestic equilibrium price, when will there be exports, imports, no trade? In each case, find the new equilibrium as well as the changes in domestic consumer and producer surplus that result from free trade.

c) In the case where the answer to b) involves imports, what will be the effect of imposing a tariff t on each unit imported? In particular, compare the tariff revenue with the change in the sum of consumer and producer surplus, and find the "deadweight loss"?

d) How do the answers to c) change if there is no tariff, but foreign suppliers are persuaded to limit their export sales "voluntarily" to the total amount E?

5. Two goods, x_1 and x_2, are perfect substitutes for person A and perfect complements for person B, the only two inhabitants of our island economy. Suppose A is endowed with 1 unit of the first and zero units of the second good while B has 9 units of the first and 10 of the second commodity.

a) Draw the appropriate Edgeworth box showing the endowments, indifference curves representing A's and B's preferences, and the area of mutual beneficial trades.

b) What will be the actual market outcome, that is what will be the equilibrium quantities and the equilibrium relative price if both A and B act as price taking utility maximizers.

c) Your generous heart tells you that this market outcome is unjust. To bring social justice to the island you consider two more equitable allocations: i) (5, 5) for person A implying the same for B and alternatively ii) (5.5, 4) for A implying (4.5, 6) for B. Which one should you choose? Does it depend on whether you are a friend of A or B?

d) What desirable welfare property does your choice from c) exhibit? Indicate graphically or mathematically all points in the Edgeworth box which exhibit this property. Is the original market outcome among them?

e) Suppose you actually wanted to achieve your choice from c). How could you redistribute the initial endowments so that the chosen allocation would result as a market equilibrium - give at least two examples and indicate the equilibrium relative price.

6. On another island Robinson Crusoe can collect $Q = a\,L^\gamma$ pounds of coconuts per day by working for a fraction L of each day (where $0 \leq L \leq 1$ and a and γ are positive constants with $\gamma < 1$). Robinson's preferences for consuming C pounds of coconuts per day and spending the fraction 1 - L of each day doing something less tedious and tiring than collecting coconuts (eg solving micro problems :) are represented by the utility function $U(C, L) = \alpha \ln C + \beta \ln(1 - L)$ with $\alpha + \beta = 1$. To help you solve this problem, let us first consider Robinson in two roles, as a consumer and as a producer.

 a) Regarding Robinson the consumer, find his utility maximizing coconut demand and labor supply.

b) Now, find the unconditional labor demand and coconut supply of Crusoe Inc., the coconut producer.

c) Finally, how many pounds of coconuts does Robinson Crusoe (healed from schizophrenia) consume, how much does he work, and what is the real wage (in pounds of coconuts per day of work).