

Homework 1

due 7/19 in class

1. Find the demand functions which subject to the budget constraint $I = p_1x_1 + p_2x_2 (+ p_3x_3)$ maximize the following utility functions:
 - a) $U(x_1, x_2, x_3) = x_1^\alpha x_2^\beta x_3^\gamma$ where $\alpha + \beta + \gamma = 1$
 - b) $U(x_1, x_2) = \min(x_1, x_2)$
 - c) $U(x_1, x_2) = x_1 + x_2$For each one also find the indirect utility function and the expenditure function. Note that you might need to be a bit creative to solve b) and c).

2. Ms. Moneypenny receives a fixed income I , faces prices p_x and p_y for the two commodities x and y , and her preferences vis-à-vis these two goods can be represented by $U(x, y) = x - (1 / y)$.
 - a) Derive her demand functions for x and y .
 - b) Determine the cross price effects, ie how does her demand for one good change when the price for the other good increases?
 - c) Illustrate this graphically and distinguish income and substitution effects.
 - d) Are x and y substitutes or complements? Which of the two effects from c) drives this result?
 - e) If you define substitutes and complements only with respect to the substitution effect then are these two goods substitutes or complements? Could this be otherwise in the two commodity case according to this definition? What about if there are more than two goods?

3. Let a consumer's preferences for all consumption bundles (x, y) satisfying $x > a$ and $y > b$ be represented by the utility function $U(x, y) = \alpha \ln(x - a) + \beta \ln(y - b)$ where α , a , β , and b are all positive parameters with $\alpha + \beta = 1$.
 - a) Given the budget constraint $I = px + qy$, find the utility maximizing demands.
 - b) Show that the expenditures px and qy are linear functions in prices and income (that's why this is called the linear expenditure system). Interpret the parameters a and b .
 - c) Show that these preferences are homothetic only if $a = b = 0$.
Now, let $a = b = 0$.
 - d) Find the indirect utility function and the expenditure function.
 - e) Find the Hicksian (= compensated) demands using Shephard's lemma, through expenditure minimization, and by using the relationship between Marshallian (= uncompensated) and Hicksian demands (all three ways please).

4. a) Suppose you have \$ 400,000 to spend on a house and "other goods." The price of 1 square foot of housing is \$ 50 and you choose to purchase your optimally sized house at 4000 square feet. Call

this house A. After you bought the house, the price of housing rises to \$ 100 per square foot. Given that you can sell house A if you want to, are you now better off or worse off? (Illustrate your answer on a graph showing your budget set before you bought the house as well as the budget set after you bought house A and the price rose.) Call the new house you would buy given your new budget set house B. Is house B smaller than house A? Does your answer depend on whether the good is normal or inferior?

b) Suppose that the price had risen before you purchased the original house. Call the house you would have bought house C. Is house C smaller than house B? Smaller than house A? Does your answer depend on whether housing is normal or inferior? Carefully justify your answer.

5. Sharon has convex preferences for consumption c and leisure l represented by $U(c, l)$. She faces a market wage of w , and a tax rate of t on labor income. Furthermore, she receives a non-wage income of R . Given child-care costs of F , Sharon prefers not to work, ie she devotes all her available time T to leisure and to caring for her child herself.

a) Illustrate this situation graphically.

The government can either give Sharon a child-care voucher worth F toward child-care or reduce her tax rate, allowing her to keep a greater percentage of her wage. Assume the government does not give up any more than it has to.

b) Under which condition does Sharon prefer one government policy over the other? Illustrate graphically.

Assume $U(c, l) = \sqrt{c} \sqrt{l}$, $t = 50\%$, and $R = F = 3 w T / 8$.

c) How much does the government have to reduce the tax rate to make her indifferent between staying home and working? In terms of w , how many hours would she work then?

6. A student with initial wealth zero has income \$ 5,000 this year, but expects to have income \$ 23,100 next year. The student can borrow or lend at the interest rate of 10 % per year. For unspecified reasons, the student intends to have zero wealth at the end of next year.

a) If the student saves all current income, how much can be spent next year?

b) If the student borrows as much as possible, how much can be spent this year if there is nothing left to spend next year?

c) Draw the student's budget constraint involving expenditure this year and next year as the two goods.

d) In the same diagram, draw the new budget constraint if the interest rate falls to 5 % per year. At what point do the two budget constraints intersect?

e) Suppose the student happens to borrow \$ 5,000 in the first year when the interest rate is 10 % per year. Show how to represent this choice by adding a suitable indifference curve to your diagram.

f) Use your diagram to represent the income and substitution effects of the decrease in the rate of interest to 5 % per year. Is it possible that the lower interest rate could lead the student to borrow less?