Economics 51
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Solution Set: Quiz 3

a. Lower left corner: Indirect Utility Function, $V(p_1, p_2, I)$
Upper right corner: $Min(p_1x_1 + p_2x_2) s.t. U(x_1, x_2) = \bar{U}$

b. There are two ways to obtain Marshallian (uncompensated) demand functions from a given expenditure function.

1. First, invert the expenditure function $E(p_1, p_2, U)$ to get the indirect utility function $V(p_1, p_2, I)$. Then use Roy's Identity to obtain the Marshallian demand function $x_i(p_1, p_2, I)$ Where:

$$-\frac{\frac{\partial V}{\partial p_i}}{\frac{\partial V}{\partial I}} = x_i(p_1, p_2, I)$$

2. First, use Sheppard's Lemma to obtain the Hicksian (compensated) demand function:

$$\frac{\partial E}{\partial p_i} = h_i(p_1, p_2, U)$$

Then, recognizing that : $x_i(p, I) = h_i\left(p, V(p, I)\right)$, replace U in the Hicksian function with the indirect utility function obtained from inverting the expenditure function (see above.)

c. We know that Sheppard's Lemma allows us to recover the Hicksian (compensated) demand function from the expenditure function:

$$\frac{\partial E}{\partial p_i} = h_i(p_1, p_2, U)$$

The elements of the Slutsky Substitution Matrix are the first partials of the hicksian demand functions. Using Sheppard's lemma, we see that this is equivalent to the second partials of the expenditure function:

$$\frac{\partial^2 E}{\partial p_i \partial p_j} = \frac{\partial h_i}{\partial p_j}$$

Why is this important? Remember that the expenditure function is concave in p. This means that the second derivative $\frac{\partial^2 E}{\partial {p_i}^2}$ is negative. From the formula above, this implies that $\frac{\partial h_i}{\partial p_i}$ is also negative. Therefore the hicksian demand function is <u>always</u> downward sloping.