

Solution Set: Quiz 2

Let  $U(x_1, x_2) = x_1 x_2^2$  and the budget constraint be  $I = p_1 x_1 + p_2 x_2$

- a. To find the demand functions we set up the Lagrangian:

$$L = x_1 x_2^2 + \lambda(I - p_1 x_1 - p_2 x_2)$$

and take first order conditions with respect to  $x_1, x_2$  and  $\lambda$

1.  $\frac{\partial L}{\partial x_1} = x_2^2 - \lambda p_1$
2.  $\frac{\partial L}{\partial x_2} = 2x_1 x_2 - \lambda p_2$
3.  $\frac{\partial L}{\partial \lambda} = I - p_1 x_1 - p_2 x_2$

From (1.) and (2.) :  $x_1 = x_2(\frac{p_2}{2p_1})$  and  $x_2 = x_1(\frac{2p_1}{p_2})$

Substituting into (3.):  $x_1 = \frac{I}{3p_1}$  and  $x_2 = \frac{2I}{3p_2}$

- b. Yes, the demand functions are both homogeneous of degree zero in  $p$  and  $Y$ . If we double both prices and income, the amount of the good demanded stays the same. More formally,  $x(tm, tp) = x(m, p)$ . Note however that the demand function is homogeneous of degree one in  $Y$  alone.  $x(tm, p) = tx(m, p)$

- c. The expenditure shares are constant.

1.  $\frac{p_1 x_1}{I} = \frac{1}{3}$
2.  $\frac{p_2 x_2}{I} = \frac{2}{3}$

- d. The indirect utility function,  $V(p, m)$  is the maximum utility attainable, given a particular  $(p, m)$ . We calculate the indirect utility function by plugging in the [Marshallian] demand functions  $x(p, m)$  into the utility function  $U(x)$ .

$$V(p, m) = \left(\frac{I}{3p_1}\right)\left(\frac{2I}{3p_2}\right)^2$$