Econ 377 University of Otago
Semester 1, 2007 Gerald Willmann

Practise Final

Please answer **ALL** questions.

Problem 1: A firm's production function is $f(x,y) = \sqrt{xy}$. It seeks to minimize its cost $wx + py$ of producing at least q units of output, but is required to use at least a units of the first input.

a) Set up the Lagrangean function and clearly list all necessary conditions.

b) Distinguishing all four possible cases, find the solution candidate (if any) for each case. Doublecheck that the candidates you find are in fact admissable.

c) Find the value function (called the cost function) and the derivatives of C^* with respect to both prices. Doublecheck your result using the envelope theorem to find the same derivatives.

Problem 2: A monopolist faces demand $D(p,\dot{p}) = -Ap + B\dot{p} + C$, has costs of $c(x) = \alpha x^2 + \beta x + \gamma$, seeks to maximize $\int_0^T [p(t)D(p,\dot{p}) - c(D(p,\dot{p}))]dt$, with $p(0)$ given and a terminal value of $p(T)$.

a) Find the Euler equation.

b) Solve for the optimal price path.

c) Check your solution using the Hamiltonian.

Problem 3: Consider the system of two differential equations $\dot{\pi}(t) = \alpha\pi(t) - \sigma(t)$ and $\dot{\sigma}(t) = \pi(t) - \sigma(t)/\beta$.

a) Transform this system into one second-order differential in π, and also into one second-order differential equation in σ.

b) Conduct a phase diagram analysis of the system to find its stationary points. Make sure to specify their exact coordinates.

c) Find the general solution assuming $\alpha + 1/\beta > 2$.

Problem 4: Suppose we want to maximize $\sum_{t=0}^{t\in\{T,\infty\}} \beta^t \ln u_t$, $\beta \in (0,1)$, and the state evolves according to $x_{t+1} = Ax_t^\alpha - u_t$, starting from x_0 at time zero, with $\alpha \in (0,1)$ and $A > 0$.

a) Write down the fundamental equation of dynamic programming, carefully indicating time subscripts and the choice variable(s) of optimization.

b) Suppose the time horizon is finite, in particular $T = 2$. Solve for the optimal consumption in each time period and the resulting capital stocks, assuming that $x_3 = 0$.

c) Suppose now that the time horizon is infinite. Verify that the optimal value function has the form $J(x) = D + E \ln x$ by solving the Bellman equation. Find the optimal policy $u_t = f(x_t)$ as well as D and E.

best of luck for the final, *Gerald*