

Assignment 8

due: Thursday, May 10, in class

Problem 1: Consider $\max \int_0^T (xu - u^2 - x^2) dt$ with $\dot{x} = u$ and $x(0) = x_0$.

- Write down the Hamiltonian and the necessary conditions.
- Solve for $p(t)$, $u(t)$, and $x(t)$, using the initial and the transversality condition.
- Check the sufficient condition.

Problem 2: Maximize $\int_0^1 (-2\dot{x} - \dot{x}^2)e^{-t/10} dt$ with $x(0) = 1$ and $x(1) = 0$.

- Solve by using the Euler equation.
- Solve by using the Hamiltonian.
- Solve by using the current value Hamiltonian.

Problem 3: A firm's production function is $Y = f(K) = K - aK^2$, its capital stock evolves according to $\dot{K} = I - \delta K$, where I is its investment and $K(0) = K_0$ is given, and its objective is to maximize the stream of profits $\int_0^T (K - aK^2 - I^2) dt$. Find its optimal investment behavior and the resulting capital stock over time.