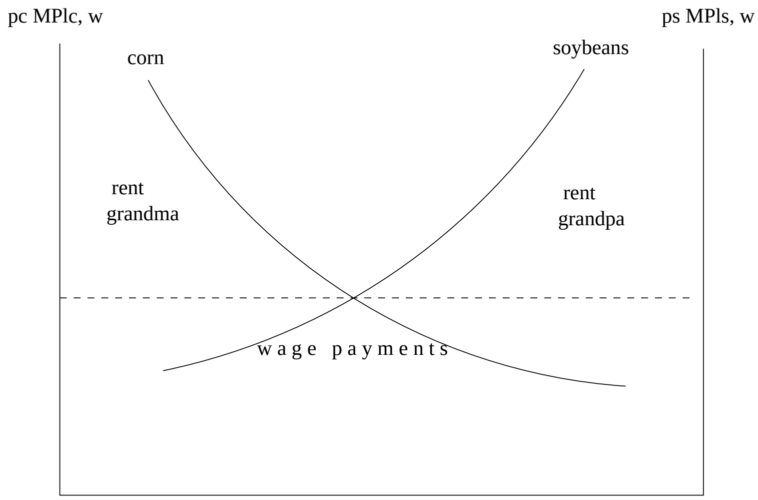
Econ 328 University of Illinois
Fall 2003/04 Gerald Willmann

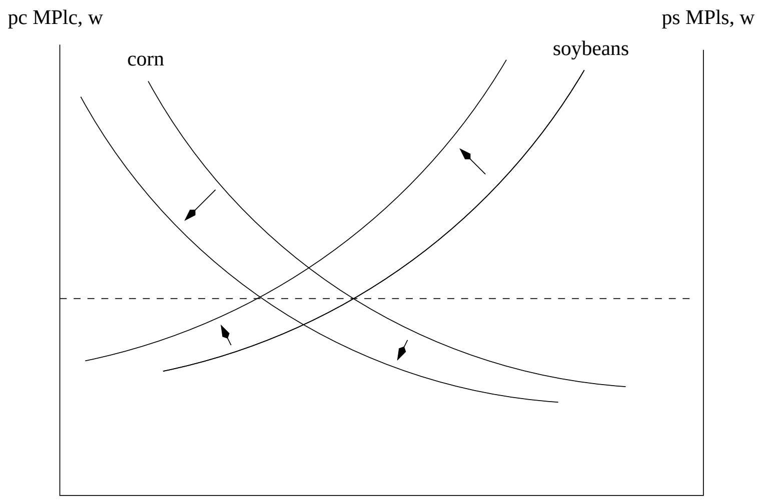
Solution HW2

Problem 1:

a) $pMP_L = w$ follows from profit maximization: $\pi = pf(L, \bar{K}) - wL \quad max!$
The resulting first order condition is $p\partial f/\partial L - w = 0$ which implies the above, noting that MP_L is the partial derivative of output with respect to labor.



b) (the following is just one example of a relative price decrease of corn vs. soy)



grandma: she experiences a decrease in real income as her sector shrinks.

grandpa: his real income increases as the soy sector expands.

for the mobile factor it depends on the consumption pattern. The nominal wage remains unchanged (in our example) so if they consume mostly corn (which has become cheaper) then their real income increases and vice versa if they mainly consume soybeans.

c) HO/Stolper–Samuelson: 2 perfectly mobile factors, one loses, the other gains.

   Specific factors model: one factor perfectly mobile, one (in each sector) perfectly immobile, of the latter one loses, the other gains, and for the mobile factor we cannot say.

   Connection: the degree of mobility is a question of time. In the short run, factors tend to be immobile, in the long run they are rather mobile. So HO/Stolper–Samuelson corresponds to the short-run, the specific factors model to the long run.

Problem 2:

a) Let us start with a general linear demand function, $Q = a - bP$ or (in inverse form) $P = a/b - Q/b$. Revenue is $QP = Qa/b - Q^2/b$ and marginal revenue $MR = a/b - 2Q/b = P - Q/b$.

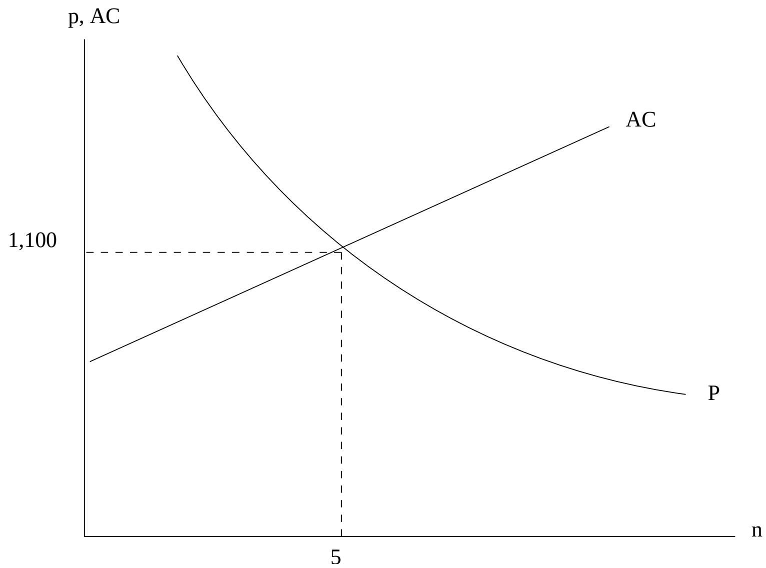
   Note that the demand function at hand, namely $Q = S(1/n - (P - \bar{P})/500)$, is linear in P with a slope of $-S/500$. So marginal revenue is $MR = P - 500Q/S$.

   Each company behaves as a monopolist setting $MR = MC$, in other words $P - 500Q/S = 1,000$, or — solving for the price: $P = 1,000 + 500Q/S$.

   Assume now that companies all behave the same way since they are all equal. In particular, they will all produce the same quantity Q and thus the total quantity traded is divided equally between them: $S = nQ$.

   Plugging this in we have $P = 1,000 + 500/n$.

b) Average cost is total cost of a firm divided by output: $AC = (F + cQ)/Q = F/Q + c$. Using symmetry as above $AC = nF/S + c = 10,000,000n/500,000 + 1,000 = 20n + 1,000$.



Setting $P = 1,000 + 500/n = AC = 20n + 1000$ gives the equilibrium quantity: $n = 5$ and plugging back into AC or the pricing equation gives a price of 1100 in equilibrium.

c) Free trade between 4 countries of equal size. The only thing that changes is that now the total market size is $S = 4 \times 500,000 = 2,000,000$.

Thus $AC = Fn/S + c = 5n + 1,000$.

Setting $P = AC$ gives $n = 10$ and $P = 1,050 < 1,100$.

We see that the number of companies per country has decreased from 5 to 2.5 and the efficiency gains of their increased scale translate into a lower price, implying gains from trade for the consumer. And since on the production side profits are still zero, we do have gains from trade for each country as a whole.