Solution HW1

Problem 1: You were given:

	C-town	U-town
labor	$L^C = 60$	$L^U = 40$
food	$a_f^C = 3$	$a_f^U = 10$
haircuts	$a_h^C = 6$	$a_h^U = 5$

Also, from $U(Q_f, Q_h) = Q_f \times Q_h$ it follows via utility maximization that $MRS \equiv \frac{\partial U/\partial Q_h}{\partial U/\partial Q_f} = Q_f/Q_h = p_h/p_f$ (*).

a) The PPFs for each town:
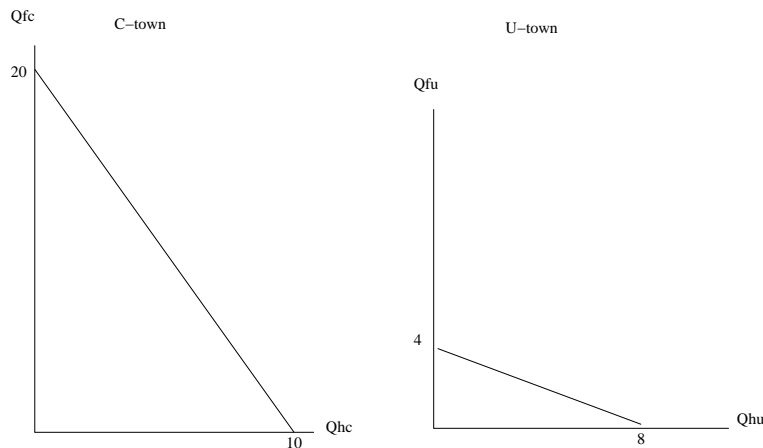
and the slopes:

$$a_{L,h}^c/a_{L,f}^c = 6/3 = 2 > a_{L,h}^u/a_{L,f}^u = 5/10 = 0.5$$

U-town has the comparative advantage in cutting hair and C-town in producing food.

b) Under autarky the relative price will coincide with the slope of the PPF, i.e. 2 in Champaign and 1/2 in Urbana. It follows from eqtn (*) above that C-tonians want twice as many units of food as haircuts and U-tonions twice as many haircuts as units of food. The point on each town's PPF that fulfills these tastes is $(Q_h^c = 5, Q_f^c = 10)$ in C-town and $(Q_h^c = 4, Q_f^u = 2)$ in U-town. The resulting utility levels are 50 in C-town and 8 in U-town (only meant to be compared to the values we obtain below).

c) Eqtn (*) can be rewritten as $p_h/p_f = \frac{1}{Q_h/Q_f}$. This is the relative demand curve we see in the diagram below — it might help to think of it as $y = f(x) = 1/x$.
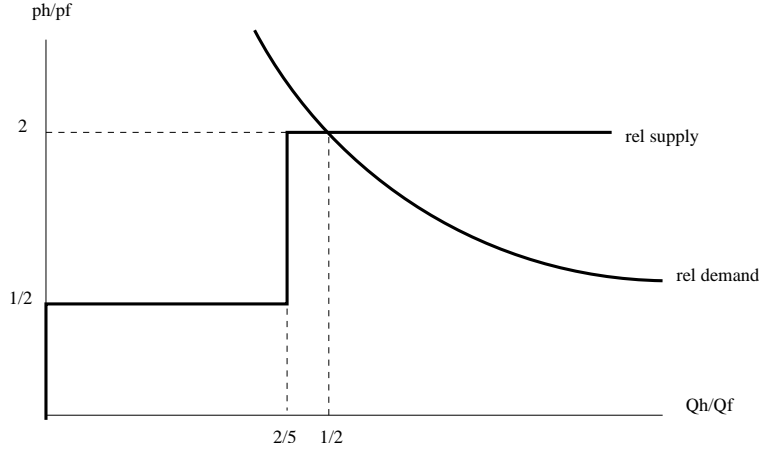
We see that the equilibrium relative price is $p_h/p_f = 2$
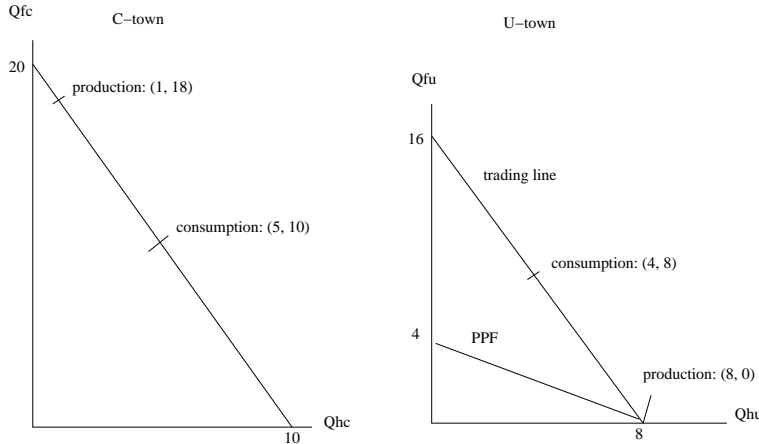
In U-town, the slope of the trading line originating at the production point $(8, 0)$ is the free trade equilibrium relative price 2. Facing this price, U-tonians also want twice as many units of food as haircuts. The optimal consumption point on the trading line is thus $(4, 8)$, giving rise to a utility of $U = 4 \times 8 = 32 > 8$. That is, U-town experiences gains from trade. The difference between consumption and production must be traded: U-town exports 4 haircuts and imports 8 units of food.

In C-town the price doesn't change as we move from autarky to free trade. In other words, the trading line coincides with the PPF. Therefore the optimal consumption point is still $(5, 10)$ as we found under b) giving rise to the same utility as under b). There are no gains from trade for C-town. Production is profit maximizing everywhere on the PPF, it therefore adjusts to accommodate U-town's trade vector. In other words, C-town imports 4 haircuts, exports 8 units of food, and correspondingly must be producing at $(1, 18)$.

Problem 2:

a)
- Stolper Samuelson: *If the price of an output good rises relative to the other output good then the factor that is used intensively in the production of the former increases in price relative to the other factor.*
- Factor Price Equalization: *Free trade equalizes factor prices in both countries.*
- Heckscher-Ohlin: *A country exports that commodity which is produced using the factor intensively with which the country is relatively well endowed.*
- Rybczynski: *An increase in the relative supply of a factor will increase the relative production of the commodity that uses this factor intensively.*

b) Cf. the handout on the Heckscher-Ohlin model.

c) No, we don't really expect the same wage in every country. One explanation for this deviation from FPE is that the assumption of identical technology does not hold. Another explanation is that a country specializes completely in which case FPE breaks down. Note that this latter possibility qualifies the FPE result as it does not necessarily hold even if all assumptions of the HO model are fulfilled.