Problem Set 4

due: Monday, March 15, *in class*

Problem 1

For each production function below, show if it has increasing, constant or decreasing returns to scale, show if the marginal products of labor and capital (MP_L and MP_K) are increasing, constant or decreasing as the amount of only the factor in question is varied. Finally, calculate the $MRTS_{L,K}$ for each prodcution function.

(a) $f(L, K) = L + 2K$

(b) $f(L, K) = \frac{1}{5}LK^2$

(c) $f(L, K) = L^{\frac{1}{4}}K^{\frac{3}{4}}$

(d) $f(L, K) = \left(L^{\frac{1}{3}} + K^{\frac{1}{3}}\right)^3$

Problem 2

Suppose that a firm's production function is given by a Cobb-Douglas function

$$Q = AL^\alpha K^\beta,$$

(where $\alpha, \beta > 0$), and that the firm can purchase all the L and K it wants in competitive input markets at rental rates w and v, respectively.

(a) Show that cost minimization requires
$$\frac{wL}{\alpha} = \frac{vK}{\beta}.$$
    What is the shape of the expansion path for this firm?

(b) Now assume that $A = 2, \alpha = \beta = \frac{1}{2}$, and that capital is fixed at $K = \bar{K}$ in the short run. Calculate the firm's total costs as a function of Q, w, v, and $\bar{K}$.

(c) Given Q, v, and w, how should the capital be chosen to minimize total cost?

(d) Use your results from part (c) to calculate the long-run total cost of production.

Problem 3

Consider the production function $Q = L^{1/3}K^{1/3}$. Calculate the profit maximizing, unconditional factor demands and the output supply as functions of the input prices w, r, and the output price p. Do so

(a) by substituting the production function into the objective,

(b) by using the Lagrangean approach,

(c) by using the (long-run) cost function derived in problem 2.

(d) Which approach do you prefer?

Problem 4

Consider a company that has two production facilities: Factory A produces with production technology $Q_A = \sqrt{L_A}$ and factory B produces $Q_B = 2\sqrt{L_B}$. Denote the output price by p and the wage rate by w.

(a) Calculate the cost functions and output supply functions for A and B as if they were separate companies.

(b) Derive the cost function and output supply function of the (combined) company.

(c) What relationship do you see between the output supply functions from a) and b)? The same relationship does not seem to hold for the cost functions. Explain intuitively how the (combined) company allocates production across its two facilities in order to minimize cost.