

Practise Final

Question 1: Suppose a person's preferences for goods 1 and 2 are given by $U(x_1, x_2) = \sqrt{x_1} + \sqrt{x_2}$. The person has an income of I , and denote prices by p_1 and p_2 .

- Find the demand functions for goods 1 and 2.
- Find the sign of the own price effect.
- Graphically decompose the effect of an increase in the price of the first commodity into income and substitution effect.

Question 2 Suppose you observe someone make the following choices:

- She buys bundle $A = (X^A = 7, Y^A = 4)$ when prices are $p_x = 3$ and $p_y = 3$;
- She buys bundle $B = (X^B = 6, Y^B = 6)$ when prices are $p_x = 4$ and $p_y = 2$;
- She buys bundle $C = (X^C = 7, Y^C = 3)$ when prices are $p_x = 5$ and $p_y = 1$.

What can you deduce about her preferences regarding bundles A, B, and C from:

- the pair of observations i) and ii);
- the pair of observations i) and iii);
- the pair of observations ii) and iii);
- all three observations taken together.

Question 3: Consider the production function $F(K, L) = K^{1/4}L^{1/2}$, and denote factor prices by r and w .

- Does this technology exhibit diminishing marginal products? Does it exhibit decreasing, increasing, or constant returns to scale?
- Derive the cost-minimal conditional factor demands (as functions of the factor prices and a given output level) and find the cost function.
- Find the profit maximizing output supply function as well as the unconditional factor demands.

Question 4: Monopoly Inc. faces demand of the form $Q = 100 - P$ and its cost function is $C(Q) = Q^2/4$.

- Find the profit maximizing output quantity and the price Monopoly Inc. charges? How much profit does it make?
- Compare welfare (consumer surplus, profit, deadweight loss) under the monopoly solution to the competitive outcome.
- Monopoly Inc. considers a two part pricing scheme: in addition to a price per unit it will also charge a fixed fee. Discuss the welfare effects of allowing Monopoly Inc. to do so.

Question 5: Pat and Eric have registered for UIUC's new island survival class. UIUC provides each of them with 10 gallons of water and 10 gallons of Coke. Pat can't stand Coke and only cares for water. Eric on the other hand regards both as perfect substitutes.

- Draw an appropriate Edgeworth box and depict a few of their respective indifference curves, the endowment point, the area of mutually beneficial trades, and the contract curve.
- Given the above endowments, find all possible market equilibria.
- State Walras' law and show it for this economy.

Question 6: Two friends, Giovanni and Cristian, play the following game. Giovanni moves first and he can either choose to go left or right. If Giovanni goes right, Cristian won't have a chance to play. Giovanni's payoff is going to be 1, while Cristian's 0. If Giovanni goes left, Cristian will play the game and he can either choose to play left or right. If he plays right, his payoff will be one, while Giovanni's will be 0. If he plays left, they will both receive a payoff of 2.

- Draw the game tree and also represent this game in a table.
- Are there any dominated strategies?
- Find the Nash equilibria of the game.

Question 7: One of the first applications of game theory is the study by Auguste Cournot of the behavior of firms producing mineral water in France around 1830. Suppose there are n companies selling mineral water, each one of them producing a quantity q_i . Let $Q = q_1 + \dots + q_n$ denote the aggregate supply on the market. Let P denote the market-clearing price and assume that the inverse demand is given by $P(Q) = a - Q$ (assuming $Q < a$, else $P = 0$). Assume that the total cost of firm i producing quantity q_i is $C_i(q_i) = cq_i$. That is, there are no fixed costs and the marginal cost is constant at c , where we assume $c < a$. Following Cournot, suppose that the firms choose their quantities simultaneously.

- What is the Nash equilibrium?
- What happens if n approaches infinity?
- Consider now the situation where there are only two firms producing mineral water and assume that the firms have asymmetric marginal costs: c_1 for firm 1 and c_2 for firm 2. What is the Nash equilibrium if $0 < c_i < a/2$ for each firm? What if $c_1 < c_2 < a$ but $2c_2 > a + c_1$?