Economics 300 | Parviainen/Willmann
Sections 2, 3, and 4 | Spring 2004

Answers to Midterm 1

1 Utility maximization

(a) When consumer's utility can be described with function $U(j,b) = \min\{2j, b\}$, the goods in question are perfect complements. Hence, the indifference curves L-shaped, and the corner point is determined by $b = 2j$.

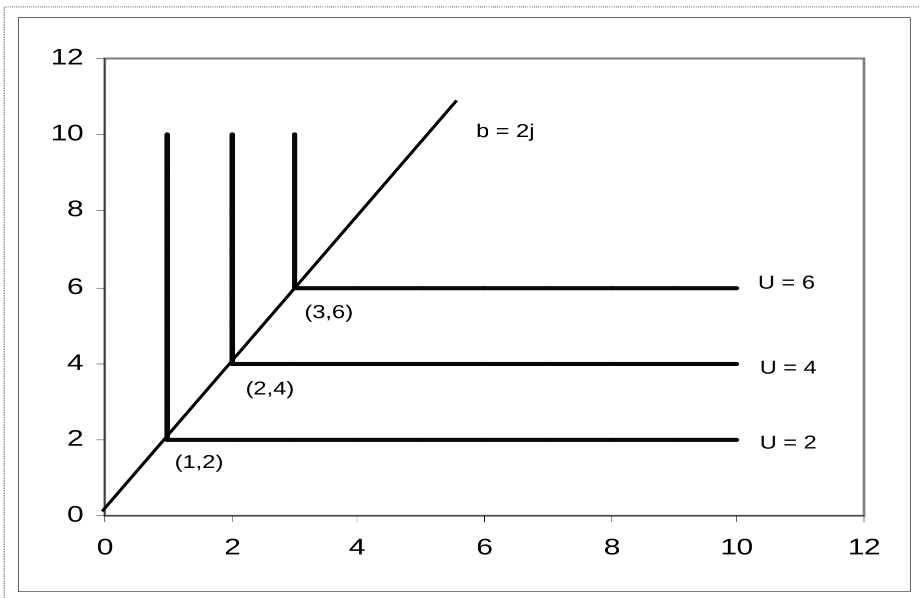


Figure 1: Leontief Preferences

(b) To solve for the optimal consumption, we need to use the optimality condition from above, $b = 2j$, and the budget constraint $P_j j + P_b b = I$. This gives

$$P_j j + P_b(2j) = I \Longrightarrow j(P_j + 2P_b) = I$$
$$\Longrightarrow j = \frac{I}{P_j + 2P_b} \quad \text{and}$$
$$b = 2j \Longrightarrow b = \frac{2I}{P_j + 2P_b} = \frac{I}{\frac{1}{2}P_j + P_b}.$$

When $I = 16, P_j = 2,$ and $P_b = 1$

$$j = \frac{16}{2+2} = 4 \quad \text{and} \quad b = \frac{16}{1+1} = 8.$$

(c) When $P_j = 3$

$$j = \frac{16}{3+2} = 3\frac{1}{5} = 3.2 \quad \text{and} \quad b = \frac{32}{3+2} = 6\frac{2}{5} = 6.4.$$

(d) When the goods are perfect complements, the substitution effect of a price change is zero. The income effect is equal to the total change.

I.e. Total change: $j_{new} - j^* = 3.2 - 4 = -0.8$

Income effect $= -0.8$

Substitution effect $= 0$.

2 Income and substitution effects

We have

$$I = 16, \quad P_x = P_y = 1, \quad U(x,y) = xy,$$
$$x(P_x, P_y, I) = \frac{I}{2P_x} \quad \text{and} \quad y(P_x, P_y, I) = \frac{I}{2P_y}.$$

(a) Optimal consumption bundle:

$$x^* = \frac{16}{2 \times 1} = 8 \quad \text{and} \quad y^* = \frac{16}{2 \times 1} = 8.$$

Original utility:

$$U(x^*, y^*) = 8 \times 8 = 64.$$

(b) Now $P_x = 4$. The new consumption bundle is

$$x' = \frac{16}{2 \times 4} = 2 \quad \text{and} \quad y' = \frac{16}{2 \times 1} = 8.$$

With the new prices, the amount of income needed to reach the original level of utility is

$$\frac{I}{2 \times 4} \times \frac{I}{2 \times 1} = 64 \Longrightarrow I^2 = 16 \times 64$$
$$\Longrightarrow I = \sqrt{16}\sqrt{64} = 4 \times 8 = 32.$$

If the consumer had $I = 32$ with prices $P_x = 4$ and $P_y = 1$, the optimal consumption point would be

$$\hat{x} = \frac{32}{2 \times 4} = 4 \quad \text{and} \quad \hat{y} = \frac{32}{2 \times 1} = 16.$$

(c) The substitution effect of the price change is $\hat{x} - x^* = 4 - 8 = -4$, and the income effect is $x' - \hat{x} = 2 - 4 = -2$.

3 Quasi-linearity

You are given $U(x, y) = x + \ln(y)$.

a) The marginal rate of substitution is $MRS \equiv \frac{MU_X}{MU_Y} \equiv \frac{\partial U/\partial X}{\partial U/\partial X} = \frac{1}{1/y} = y$.
Why is it equal to the relative price? Write down the Lagrangean:

$$maxL = U(X, Y) - \lambda(P_X X + P_Y Y - I)$$

Calculate the first order conditions with respect to X and Y:

$$\begin{aligned} \frac{\partial U(X,Y)}{\partial X} - \lambda P_X &= 0 \\ \frac{\partial U(X,Y)}{\partial Y} - \lambda P_Y &= 0. \end{aligned}$$

Bringing the respective second terms over to the other side, and then dividing one equation by the other yields the above result.

More intuitively, this is the tangency condition that the slope of the budget constraint (the relative price) must equal the slope of the indifference curve (the MRS) because otherwise there would be scope for improvement.

b) From a) we have

$$Y = \frac{P_X}{P_Y}$$

This is the demand function for good Y. Plug it into the budget constraint and we obtain the demand function for good X:

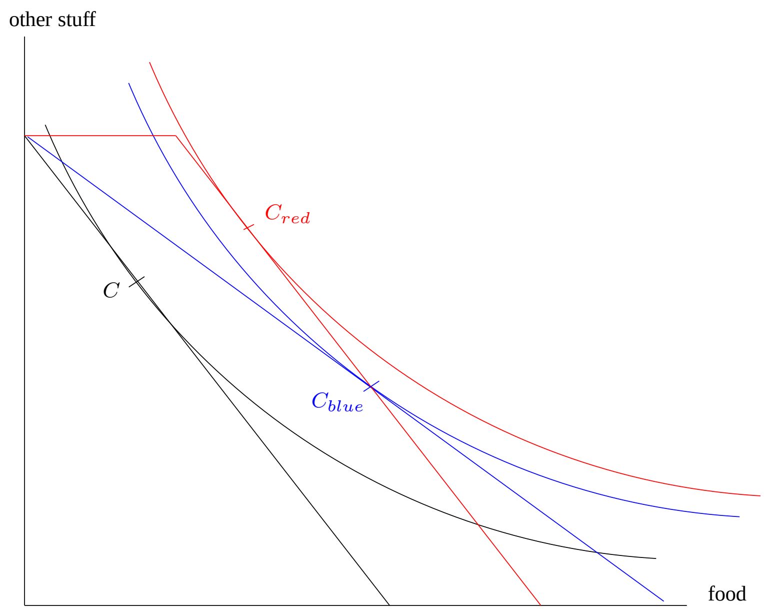
$$X^D(p, I) = \frac{I}{P_X} - 1$$

c)

$$\epsilon_{X,I} \equiv \frac{\partial X^D}{\partial I} \times \frac{I}{X^D(I)} = \frac{1}{P_X} \times \frac{I}{\frac{I}{P_X} - 1} = \frac{I}{I - P_X}$$

Assuming that income exceeds the price of X (otherwise we couldn't be spending P_X on good Y), this income elasticity is greater than one. It must be, because as income rises our consumption of Y remains unchanged so we must be spending an ever increasing percentage of our income on X.

4 Food and other Goods



a) The blue party introduces a price subsidy that shift's out the budget line as depicted in the diagram. The price of food is lower under this scheme corresponding to the shallower slope. The demand for food increases as the income effect is most likely positive (or do you spend more on food than Bill Gates or Jacques Chirac?).

b) The red party's free food is represented by the horizontal segment. If you want more, then you pay the regular price, i.e. the slope is the same as for the origional black budget line. Demand increases as long as the income effect is positive. Since there is no substitution effect this scheme increases demand by less than the blue scheme.

c) Let us compare both schemes. The assumption that both parties use the same amount of funding implies that the red budget line must go through the blue consumption point. But since it has a steeper slope this implies that under code red you are better off than under code blue (not to mention the reduced risk of obesity). So the econ-savvy voter votes red.