Econ 300 University of Illinois
Spring 2003/04 Gerald Willmann

Homework 6

due: Monday, May 3, in class

Problem 1: Consider an exchange economy with two agents, A and B, and two commodities, 1 and 2. The respective utility functions are $U^A(x_1, x_2) = (x_1 x_2)^2$ and $U^B(x_1, x_2) = \ln x_1 + 2 \ln x_2$. The endowments are $e^A = (18, 4)$ and $e^B = (3, 6)$. Let prices be denoted by p_1 and p_2.

a) For each agent, A and B, write down their budget constraint and derive their demand functions.

b) Use the market clearing condition(s) to find the market equilibrium, i.e. the equilibrium relative price and the equilibrium qunatities.

c) Graphically depict your solution in an Edgeworth-box. Explain why and in which sense the market equilibrium from b) is efficient.

Problem 2: Again, consider an exchange economy populated by George and Laura. Both of them have identical preferences which can be represented by $U(x_1, x_2) = x_1 - 1/x_2$, take prices as given, and have endowment vectors $(e_1^G = 8, e_2^G = 0)$ and $(e_1^L = 0, e_2^L = 8)$ respectively.

a) Draw the corresponding Edgeworth box, indicate the endowment point, sketch a few indifference curves, and depict the contract curve.

b) Calculate the market outcome, i.e. equilibrium price and quantities.

c) State Walras' law and verify it for this economy.

Problem 3: Consider the following two games between a seller who has to decide whether to offer high or low quality and a buyer who has to decide simultaneously whether to buy:

game A	high	low
buy	6, 10	-2, 4
not	0, 2	0, 0

game B	high	low
buy	6, 4	-2, 10
not	0, 0	0, 2

(where the first number indicates the buyer's pay-off and the second the seller's)

a) In each game and for each player's actions, find the best response of her opponent.

b) Do the buyer and/or the seller have a dominant strategy in either game? Indicate the dominant strategies if there are any.

c) Find the Nash-equilibrium for both games.

Problem 4: Consider a market with inverse demand function $P = 120 - Q$ and two companies that engage in quantity competition (Cournot duopoly). The cost function of the first company is $C_1(Q_1) = 20Q_1$ and for the second it is $C_2(Q_2) = 10Q_2$.

a) Find the best response functions of both companies.

b) Find the Nash-equilibrium. Also calculate the market price and profits in this equilibrium.

c) Suppose company 1 gets to move first (Stackelberg leader). What quantity will company 1 set, taking into account the response of the second company? What is the resulting price and the profits in this equilibrium?