

## Econ 300 - Solution to Homework 5

### Question 1

a)  $p = 102 - Q$

$$\pi = Q(102 - Q) - (102 + 0.5 Q^2)$$

$$\text{FOC: } \frac{d\pi}{dQ} = 102 - 3Q = 0$$

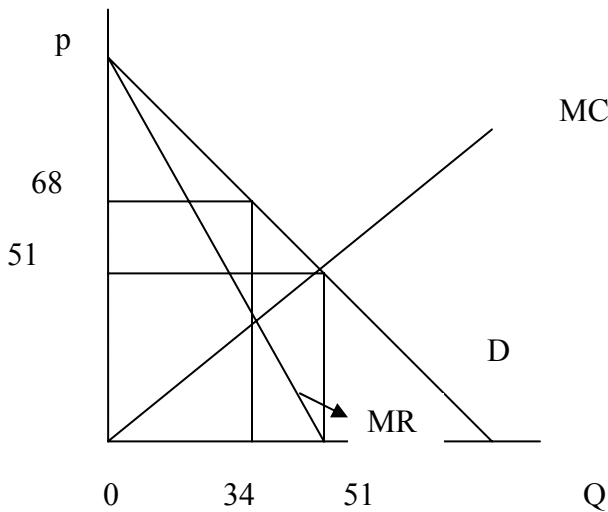
Then  $Q^* = 34$ ,  $p = 68$  and Revenue =  $34 \times 68 = 2312$   
Cost =  $102 + 0.5 \times 34^2 = 680$  and Profit =  $\pi^* = 2312 - 680 = 1632$

b) In this case  $102 - Q = Q$  then  $Q = 51 = p$

$$\text{Revenue} = 51 \times 51 = 2601$$

$$\text{Cost} = 102 + 0.5 \times 51^2$$

Profit =  $\pi^* = 2601 - 1402.5 = 1198.5$ , so profit is lower in this case



Consequences for welfare: consider the consumer surplus. Under monopoly is

$$\int_{68}^{102} (102 - p) dp = [102p - 0.5p^2]_{68}^{102} = 5202 - 4624 = 578$$

Under competition is:

$$\int_{51}^{102} (102 - p) dp = [102p - 0.5p^2]_{51}^{102} = 0.5 \times 51^2 = 1300.5$$

As we can see the consumer surplus is higher under competition. Profits are lower so the consequences for welfare are:

$$W^m = \pi^m + CS^m = 2210$$

$$W^c = \pi^c + CS^c = 2499$$

Where m denotes monopoly and c competition

So we see that welfare has increased under competition because the rise in consumer surplus outweighs the loss in profits. The deadweight loss of monopoly is the difference in welfare, that is 289.

- c) In case (a) the solution does not change. For the monopoly, maximizing  $\frac{1}{2}$  profits gives the same FOC and so the same solution as before.

In case (b):

$$p = 102 - Q - 6$$

$$\pi = Q(102 - Q - 6) - (102 + 0.5Q^2)$$

$$\text{FOC: } \frac{d\pi}{dQ} = 96 - 3Q = 0$$

Then  $Q^* = 32$ ,  $p = 70$  (after tax) and 64 (before tax)

$$\text{Revenue} = 32 \times 64 = 2048$$

$$\text{Cost} = 102 + 0.5 \times 32^2 \text{ and Profit} = \pi^* = 2048 - 614 = 1434$$

So the profit here is lower because of the tax.

In case (c)  $p = 50$ . If the original price were below 50 the ceiling would have had no effect, but since the ceiling is below the monopoly price (68) the monopoly is forced to fixed  $p = 50$ .

$$\text{Then } Q = 52 \text{ and profits are } 50 \times 52 - (102 + 0.5 \times 52^2) = 1146.$$

In this case the monopolist is worse because of lower profits, the government is worse off because its revenue has declined but the consumers are better off because of the lower price.

### Question 2

$$\pi = pQ - Q_a^2 - 2Q_b^2 \quad \text{s.t. } Q = Q_a + Q_b$$

$$\text{So } \pi = p(Q_a + Q_b) - Q_a^2 - 2Q_b^2$$

FOC:

$$\frac{d\pi}{dQ_a} = p - 2Q_a = 0$$

$$\frac{d\pi}{dQ_b} = p - 4Q_b = 0$$

$$\text{So } Q_a = p/2, Q_b = p/4 \text{ and } Q = p^{3/4}$$

$$\text{b) } \pi = p(Q_a + Q_b) - Q_a^2 - 2Q_b^2 = (100 - Q_a - Q_b)(Q_a + Q_b) - Q_a^2 - 2Q_b^2$$

FOC:

$$Q_a: -(Q_a + Q_b) + (100 - Q_a - Q_b) - 2Q_a = 0$$

$$Q_b: -(Q_a + Q_b) + (100 - Q_a - Q_b) - 4Q_b = 0$$

Then  $Q_a = 2Q_b$  and

$$-2Q_b - Q_b + 100 - 2Q_b - Q_b - 4Q_b = 0$$

$$\text{then } Q_b = 10, Q_a = 20, Q = 30 \text{ and } p = 70$$

### Question 3

a) There are several possibilities. For example:

- There are market power on campus and competition off campus
- There are market power both on and off campus, but demand elasticity is less on campus, perhaps because students don't want to walk outside campus to buy a coffee
- Higher cost on campus (may be related to market power, see (c) below)

b) You have two data: marginal cost and elasticity of student demand. Recall the equalities:

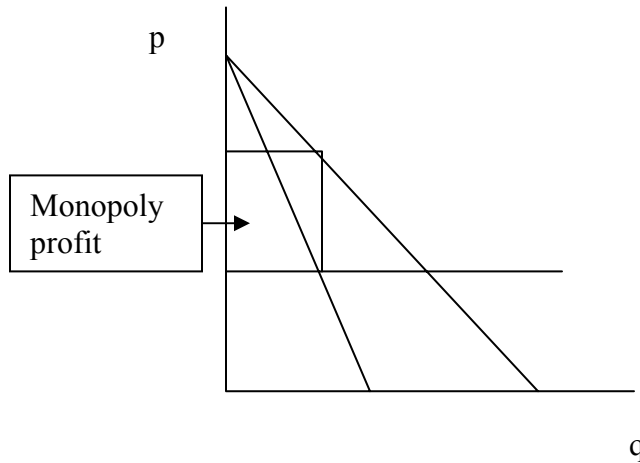
Competitive pricing:  $p = MC$

$$\text{Monopoly pricing: } \frac{d\pi}{dp_2} = 55 - (p_2 + 5)2 - 5(70 - 2p_2)p_2 - 2p_2 + 5 + 10 = 0 \quad \frac{p - MC}{p} = \frac{1}{\varepsilon}$$

where  $\varepsilon$  denotes the price elasticity of demand.

With the data you have you can test these hypothesis. If the data support the latter hypothesis you can conclude that there is monopoly power.

c) There would be a monopoly profit. In this case the consumers loose because of the higher price. The monopoly gains in principle. Now, since there is competition for the *right* to serve coffe on campus, the university may end with the monopoly rent. To see this, imagine other market, the market of righths to sell coffe on campus. There are many costumers, ie., firms that want to sell coffe on campus, and only one seller, the university, ie., the owner of the rights.



So the university, knowing that the firm which gains the rights will end up with monopoly power, can charge that firm an ammount equal to the monopoly rent. For the firm this is still profitable: if it stays away (or don't gain the rights) it's profit is 0. If it sells coffe on camus, it's profit are the monopoly profits minus the rent that it has to pay to the university. As long as this difference is greater or equal than 0, ie. As long as  $\text{Rent} \leq \text{monopoly profits}$ , the firm prefers to gain the rights (when = the firm is indifferent).

#### Question 4

- a) If the firm can sepparate markets it acts in each of them as a monopolist and charge price according to the price elasticity of demand in each market.

$$TR_1 = (55 - Q) Q$$

$$\text{Then } MR_1 = 55 - 2 Q$$

$$MR_1 = MC \text{ then } Q_1 = 25 \text{ and } p_1 = 30$$

$$TR_2 = (35 - 0.5 Q) Q$$

$$\text{Then } MR_2 = 35 - Q$$

$$MR_2 = MC \text{ then } Q_2 = 30 \text{ and } p_2 = 20$$

Now,

$$\pi = TR_1 + TR_2 - C(55) = 750 + 600 - 275 = 1075$$

b)  $p_1 = p_2 + 5$  then

$$\pi = Q_1(p_2 + 5) + Q_2 p_2 - 5(Q_1 + Q_2) =$$

$$= [55 - (p_2 + 5)](p_2 + 5) + (70 - 2p_2)p_2 - 5[55 - (p_2 + 5)] - 5(70 - 2p_2)$$

FOC:

$$\frac{d\pi}{dp_2} = 55 - (p_2 + 5) - 5(70 - 2p_2) - 2p_2 - 5 = 0$$

then,  $130 = 6p_2$  and  $p_2^* = 21.66$

then  $p_1^* = 26.66$ ,  $Q_1^* = 28.33$  and  $Q_2^* = 26.66$

Replacing these prices and quantities into the definition of  $\pi$  gives  $\pi^* = 1058.33$

c) Market demand:  $Q = Q_1 + Q_2 = 125 - 3p$

$$TR = (125 / 3 - 1/3 Q) Q$$

$$MR = 125 / 3 - 2/3 Q$$

$$\text{Making } MR = MC : 125 / 3 - 2/3 Q = 5$$

$$125 - 2Q = 15$$

$$Q^* = 55$$

Then  $p^* = 23.33$ ,  $Q_1^* = 31.67$  and  $Q_2^* = 23.33$ .

Replacing these into  $\pi$  gives  $\pi^* = 55 \times 23.33 - 55 \times 5 = 1008.31$

d) You can do that at: [www.buecker.de](http://www.buecker.de) (German online bookstore)

Besanko – Braeutigan: € 49.90 = USD 61