Econ 300, Lecture 4, G. Willman

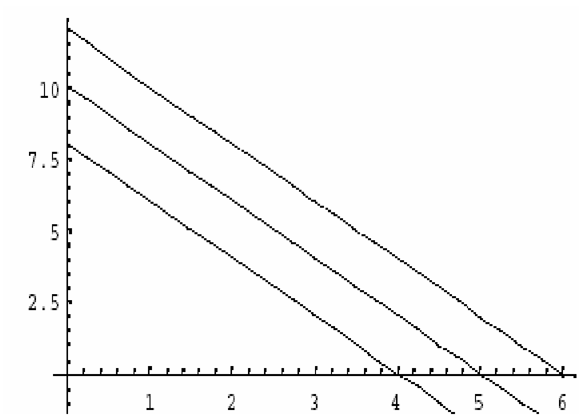
Solution to Homework 2

Problem 1

a) U (x,y) = $\alpha x + y$;   b) U (x,y) = x.y;   c) U (x,y) = ln x + ln y

a) Indifference curves: U (x,y) = A, where A is any constant

for a) $\alpha x + y = A$ then $y = A - \alpha x$. In the graph, we show indifference curves with $\alpha = 2$ and A = 12, 10, 8.



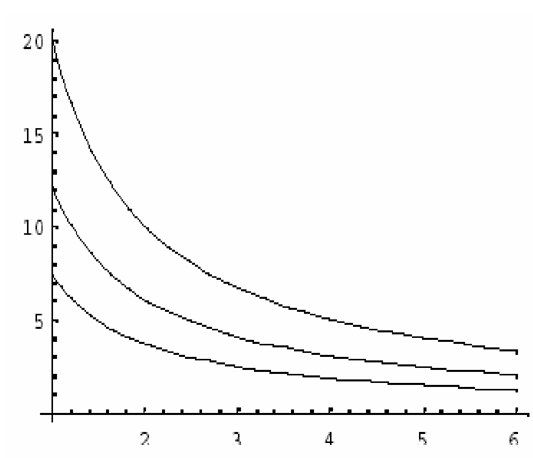
for b) x.y = A then y = A/x

for c) ln x + ln y = A then ln x.y = A

$$x.y = e^A$$

$$y = e^A / x$$

In the graph, we show indifference curves with A = 2, 2.5, 3. The graph for the two cases is similar, with only the "constant" changing.



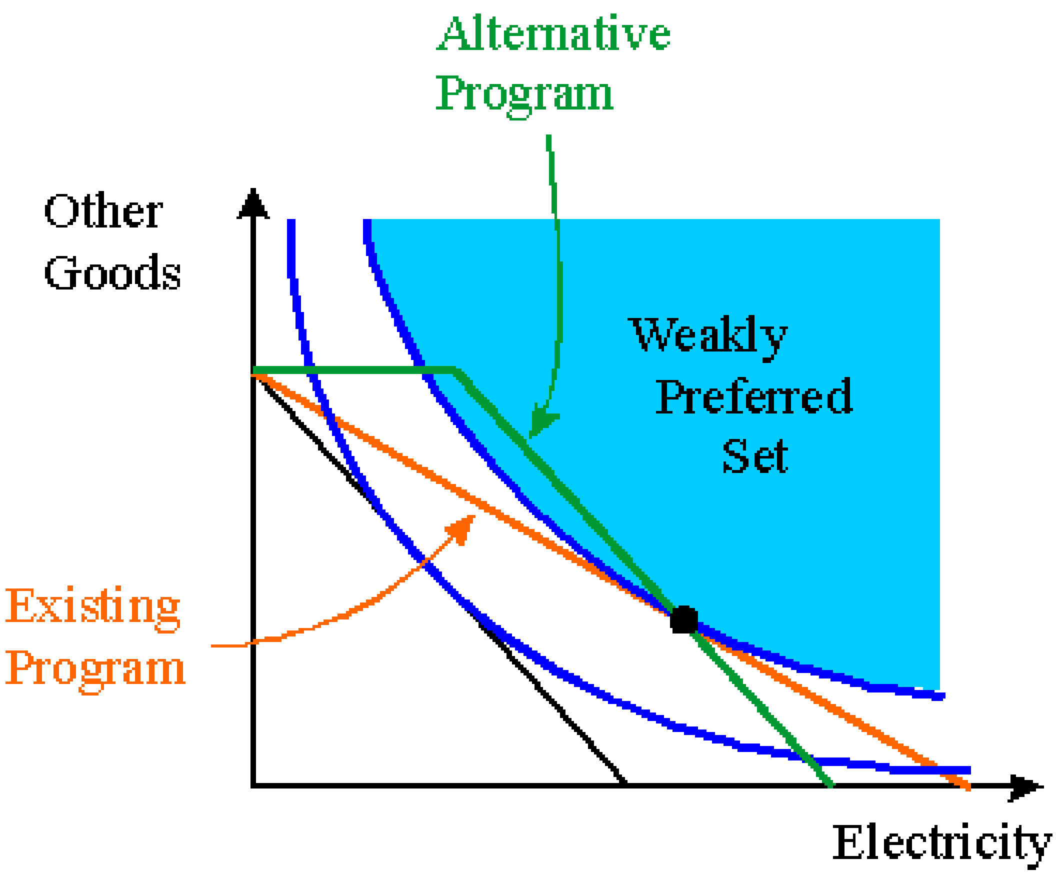
b) Example of the first utility function: a six pack of beer vs. 1 2L Coke.

c) MRS = MU_{x1} / MU_{x2}

For U1: MRS = - α / 1 = - α
For U2: MRS = - y/x
For U3: MRS = - x^{-1} / y^{-1} = - y/x

d) As can be seen from (a) the third utility function is very similar to the second, the only difference is the constant, the constant of the second utility being the power of the number e in the third one.

Problem 2



a) The initial price of electricity is p_e and the slope of the budget constraint is p_e/p_{og} where og are other goods. This is the black line in the graph. When the government establish a subsidy to the price of electricity, for the consumers the nwe price is p_e-S. Then the slope of the new budget constraint is p_e-S/p_{og} < p_e/p_{og}. If the consumers spend all their income i other goods, they can afford the same quantity than before, but if they spend all their income in electricity they can buy more now. This is reflected in the orange line.

Whit the new plan, consumers can buy electricity for free up to some point. Then, up to this point the budget constraint is parallel to the x axe. If consumers decide to buy more electricity, they should pay p_e, the original price, so from that point the slope of the budget constraint coincides with the initial one. These facts are reflected in the kinked green budget constraint.

b) To see that the "optimal" consumption of electricity has diminished, note that the new indifference curve is tangent to the green budget constraint at a point that lies to the left (measured with respect to the x axe) to the original one, meaning that the consumption of electricity has declined.

c) Who benefits? The consumers, because they can achieve a higher utility (look at the indifference curves). The government spends as much money as before, and therefore has no money to pay the economist.

Problem 3

$U(x,y) = x^a y^{(1-a)}$

Problem: Max $U(x,y)$ s.t. $I - p_x x - p_y y = 0$ then we have to

Max $x^a y^{(1-a)} + \lambda [I - p_x x - p_y y]$

FOC:

1) $ax^{(a-1)} y^{(1-a)} - \lambda p_x = 0$
2) $(1-a) x^a y^{-a} - \lambda p_y = 0$
3) $I - p_x x - p_y y = 0$

From 1 and 2: $a p_y y = (1-a) p_x x$, then $y = x p_x/p_y (1-a)/a$

Replacing the expression for y into the 3rd foc:

$I - p_x x - p_y [x p_x/p_y (1-a)/a]$,
then: $I - p_x x [1 + (1-a)/a] = 0$

$I - p_x x\, 1/a = 0$ and $x^* (I, p_x, p_y) = aI/ p_x$
$y^* (I, p_x, p_y) = (1-a)I/ p_y$

b) Expenditure shares

$p_x x / I = a$, and $p_y y / I = (1-a)$

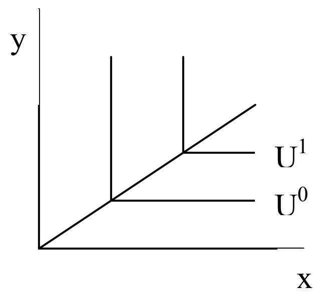
You always (independent of income level) spend a % of your income on good x and (1-a) % of your income on good y.

c) Income elasticities

$\eta_{I,x} = dx/dI \,.\, I/x = aI / p_x \,.\, p_x / aI = 1$
$\eta_{I,y} = dy/dI \,.\, I/x = (1\text{-}a)I / p_y \,.\, p_y / (1\text{-}a)I = 1$

Problem 4

Leontieff Utility



U (x,y) = min (2x,y)

b) You are going to consume so that 2x = y. Since you spend all your income:

$I = p_x x – p_y y$
$I – p_x x – p_y\, 2x$
then:
$x^* = I / (p_x + 2\, p_y)$
$y^* = I / (0.5\, p_x + p_y)$

c) Cross price elasticities

$\varepsilon_{x,py} = dx / dp_y \,.\, p_y/x = -\, 2p_y / (p_x + 2\, p_y)$
$\varepsilon_{y,px} = dy / dp_x \,.\, p_x/y = -\, 0.5p_x / (0.5p_x + p_y)$

Here, both cross price elasticities are negative because we have perfect complements.