

Homework 1

due: Wednesday, Jan 28, in class

Problem 1: Consider the following functions: $g(x) = x - x^2$, $f(x) = x^3 - 12x^2 + 36x + 8$, and $h(x) = \sqrt{x} - x$.

- Graph each function.
- Calculate the derivative of each function.
- Find the extrema and determine whether they are minima or maxima.

Problem 2: Consider the function $f(x, y) = x^2 + xy + 2y^2 + 3y$.

- Find the partial derivatives of this function with respect to x and y .
- Write down the system of (two) equations, that determines the optimum.
- Solve the system to find the optimum and determine whether it is a maximum or a minimum.

Problem 3: We want to maximize $y = -x_1^2 + 2x_1 - x_2^2 + 4x_2 + 5$ where x_1 and x_2 are constrained to add to one, i.e. $x_1 + x_2 = 1$.

- Write down the Lagrangean function for this constraint optimization.
- Find the first order conditions with respect to x_1 and x_2 and show that the first condition with respect to the multiplier is actually the constraint.
- Solve this system of (three) equations to find the optimum.

Problem 4: Consider the constrained optimization problem from the previous question.

- Use the constraint to reduce the objective function to a function of one variable only.
- Maximizing this function, find the optimal values for x_1 and x_2 .
- Show that the solution is the same as above. Which approach do you prefer?