Econ 300
Spring 2004

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Answers to Problem Set 3

1 Question 1

(a) Using the Lagrangian

$$maxL = U(F,H) - \lambda(P_F F + P_H H - I)$$

you can write down the first order conditions with respect to food and housing as

$$\frac{\partial U(F,H)}{\partial F} - \lambda P_F = 0$$
$$\frac{\partial U(F,H)}{\partial H} - \lambda P_H = 0.$$

Remember, that

$$MRS = \frac{MU_F}{MU_H}.$$

Using the first order conditions from above, you can see that

$$\begin{aligned} MRS \equiv \frac{MU_F}{MU_H} &= \frac{P_F}{P_H} \\ \Longrightarrow \frac{1/(F-C_F)}{1/(H-C_H)} &= \frac{H-C_H}{F-C_F} = \frac{P_F}{P_H} \quad \text{or} \\ P_H(H-C_H) &= P_F(F-C_F) \end{aligned}$$

(b) To find the optimal consumption point, you can use the condition derived above with the consumer's budget constraint to get

$$\begin{aligned} I &= P_F F + P_H H \\ &= P_F F + P_F(F-C_F) + P_H C_H \\ \Longrightarrow 2P_F F &= I + P_F C_F - P_H C_H \\ \Longrightarrow F(P_F, P_H, I) &= \frac{I + P_F C_F - P_H C_H}{2P_F} \\ \text{and} \quad H(P_F, P_H, I) &= \frac{I + P_H C_H - P_F C_F}{2P_H} \end{aligned}$$

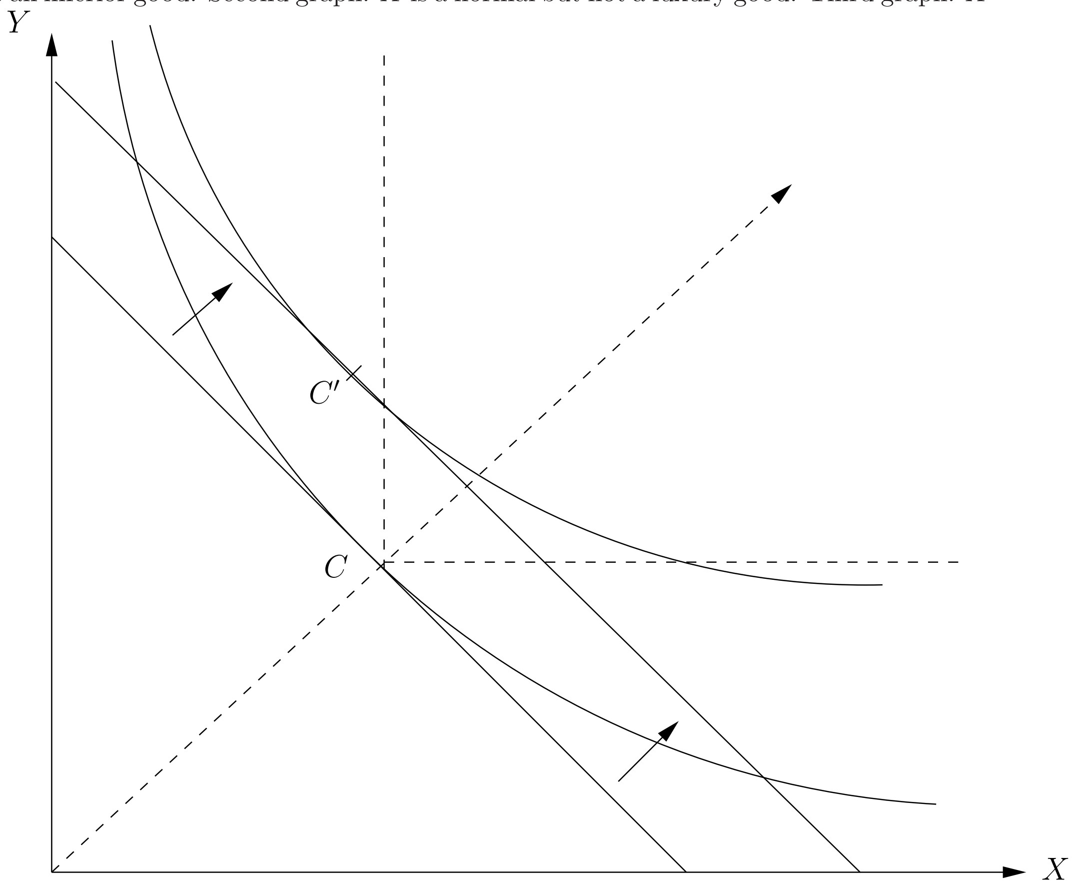
(c) Rewrite

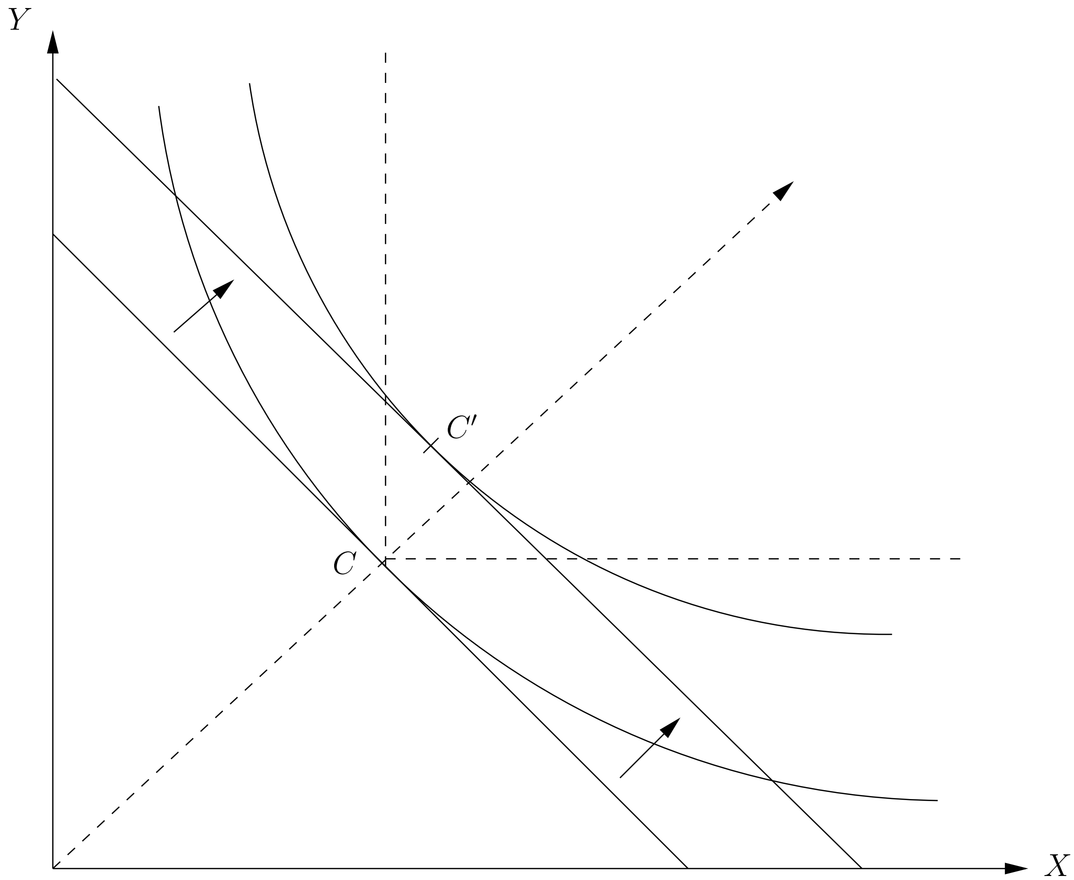
$$P_F F - P_F C_F = \frac{I - P_F C_F - P_H C_H}{2} \quad \text{and}$$
$$P_H H - P_H C_H = \frac{I - P_F C_F - P_H C_H}{2}.$$

C_F and C_H are subsistence levels. Consumption beyond these levels behaves like Cobb-Douglas, i.e. the demand for each good is a constant share of the income minus what has to be spent for survival.

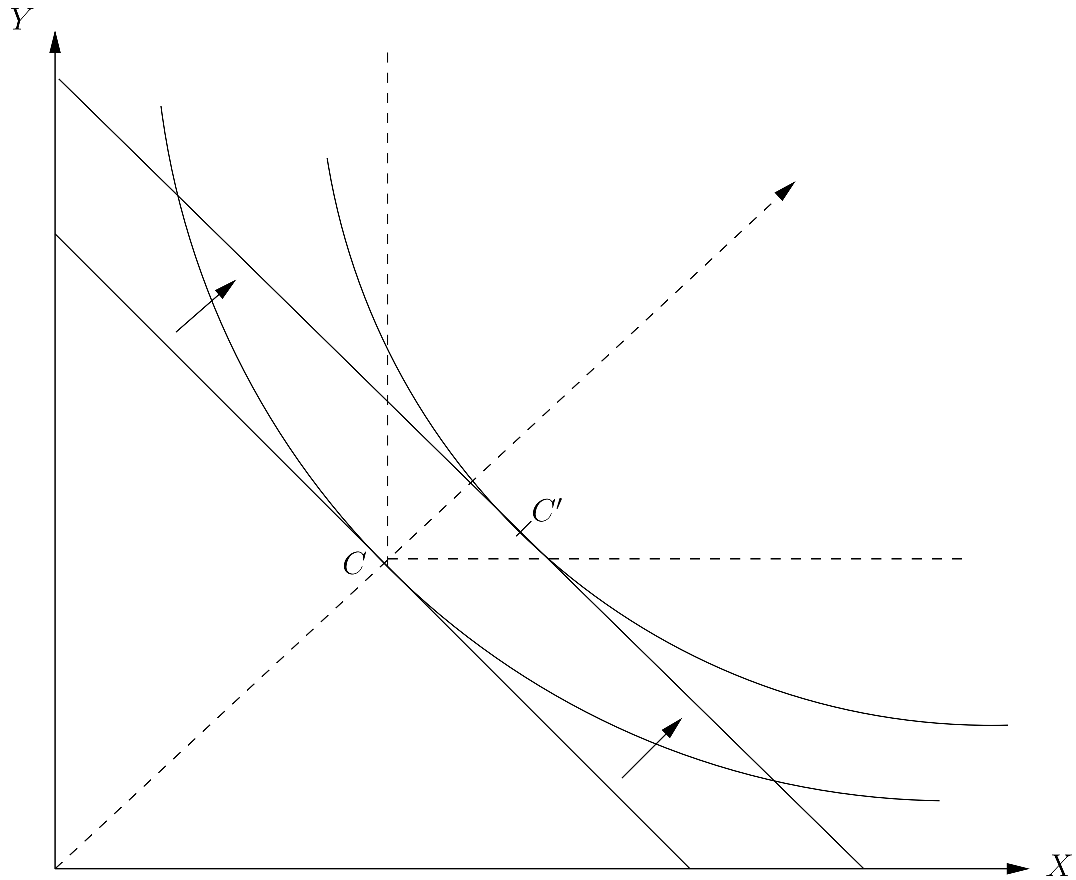
2 Question 2

(a) First graph: X is an inferior good. Second graph: X is a normal but not a luxury good. Third graph: X is a luxury good.

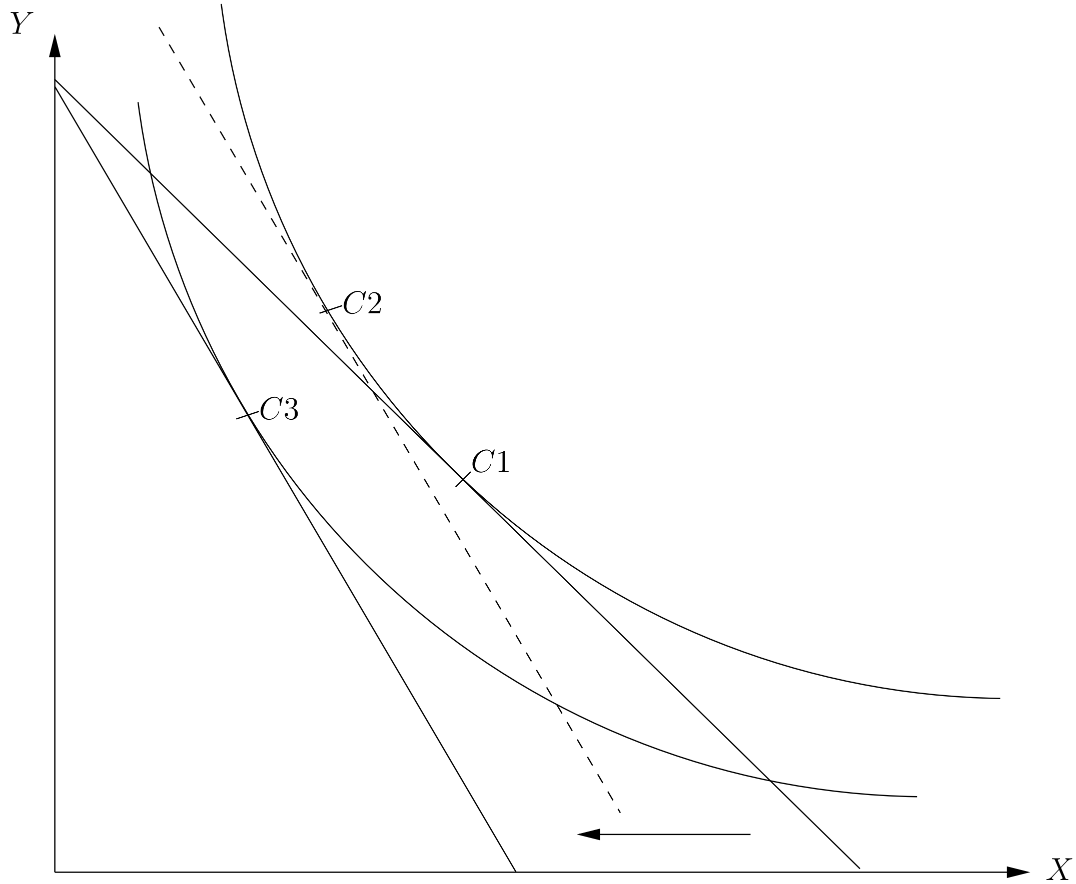




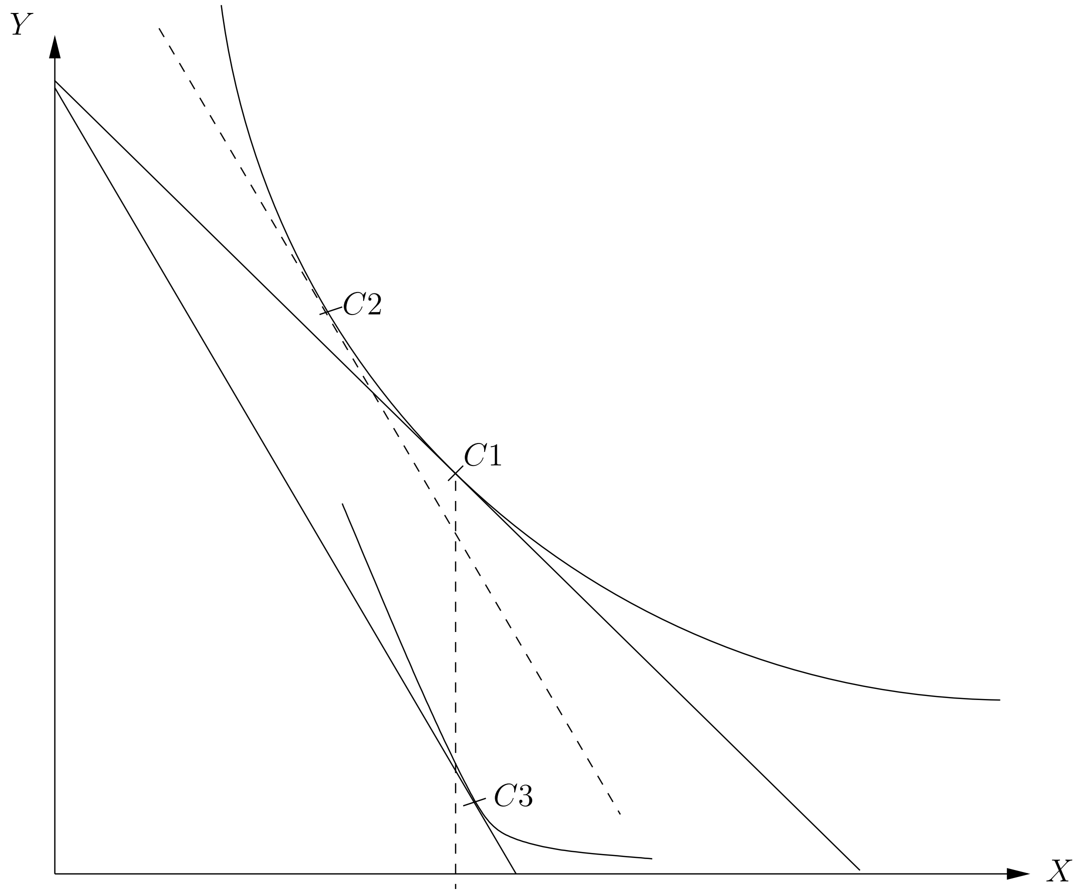
Y
C′
C
X


Y
C′
C
X

(b) For a normal good, the income effect reinforces the substitution effect.



(c) If X is a Giffen good, then a positive income effect overrides the negative substitution effect, and a price decrease will result in a dercease in consumption.



3 Question 3

(a) A lower tax rate would give rise to income and substitution effects on a person's choice of consumption and leisure. The income effect would increase both consumption and leisure, if both are normal goods, since the reduction in the tax rate leaves more after-tax income. The lower tax rate would increase the slope of the budget constraint, so the substitution effect would increase consumption and decrease leisure. The net result is an increase in consumption and an ambiguous effect on leisure, and thus an ambiguous effect on labor supply.

(b) An increase in the amount on which no tax is owed would be a pure income effect. If both consumption and leisure are normal goods, both would increase, so labor supply would decrease.

4 Question 4

(a)

$$Q_{hc}^D(P_{hc} = 3, I = 600) = 20 \quad \text{and}$$
$$Q_{hc}^D(P_{hc} = 3, I = 6,000) = 200.$$

By looking at the demands, it looks like the demand is a constant share of your income, which implies that

$$\epsilon_{Q_{hc},I} = 1.$$

Let's check if this is the case:

$$\epsilon_{Q_{hc},I} = \frac{\partial Q_{hc}}{\partial I} \times \frac{I}{Q_{hc}} = \frac{1}{10P_{hc}} \times \frac{I}{\frac{I}{10P_{hc}}} = 1.$$

(b)

$$Q_{hc}^D(P_{hc} = 5, I = 600) = 12.$$

A change in the consumer surplus, or ΔCS is

$$\begin{aligned}\int_3^5 Q_{hc}^D dP_{hc} &= \frac{I}{10}\int_3^5 \frac{1}{P_{ch}} dP_{hc} \\ &= \frac{I}{10}\left[ln(P_{hc})\right]\Big|_3^5 \\ &= 60(ln(5) - ln(3)) \approx 30.65.\end{aligned}$$

Just calculating $Q \times \Delta P$ would be wrong, because it doesn't take into account the re-optimization after a change in P.

(c) Use the demand function and the consumer's budget constraint:

$$\begin{aligned}Q_{hc}^D &= \frac{I}{10P_{hc}} \\ I &= P_{hc}\frac{I}{10P_{hc}} + P_0Q_0 \\ \implies \frac{9}{10}I &= P_0Q_0 \\ \implies Q_0^D &= \frac{9I}{10P_0}.\end{aligned}$$

This is a Cobb-Douglas utility function with $\alpha = 0.1$, hence $U = Q_{hc}^{0.1}Q_0^{0.9}$. Happiness before the price change:

$$\begin{aligned}U &= \left(\frac{I}{10P_{hc}}\right)^{0.1}\left(\frac{9I}{10P_0}\right)^{0.9} \\ &= 600\left(\frac{1}{30}\right)^{0.1}\left(\frac{9}{10P_0}\right)^{0.9} \\ &\approx \frac{600 \times 0.711685 \times 0.90953}{P_0^{0.9}} \\ &\approx \frac{388.38}{P_0^{0.9}}.\end{aligned}$$

Happiness after the price change:

$$\begin{aligned}\left(\frac{I+x}{10\times 5}\right)^{0.1}\left(\frac{9(I+x)}{10P_0}\right)^{0.9} &= \frac{388.38}{P_0^{0.9}}\\ \iff (I+x)\left(\frac{1}{50}\right)^{0.1} 0.9^{0.9} &= 388.38.\end{aligned}$$

With $I = 600$ and $P_{hc} = 5$ we get

$$\begin{aligned}(600+x)\times 0.67624\times 0.90953 &= 388.38\\ \implies x &= 31.45\end{aligned}$$