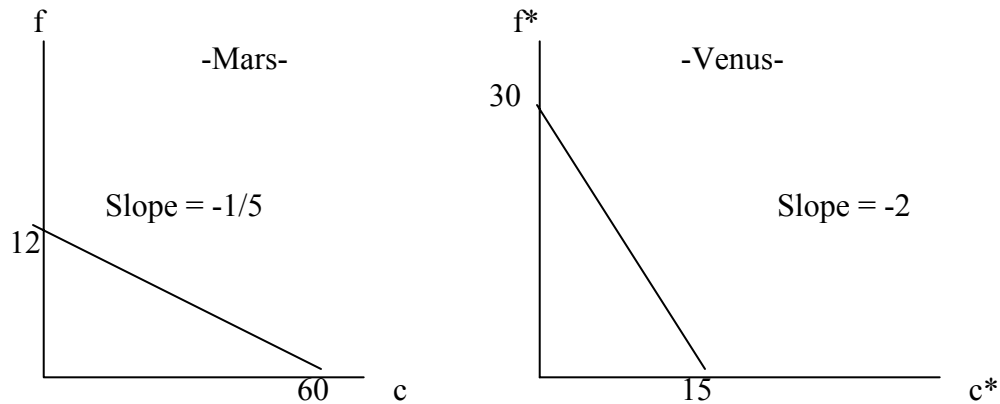


**Homework 1  
Answer Key**

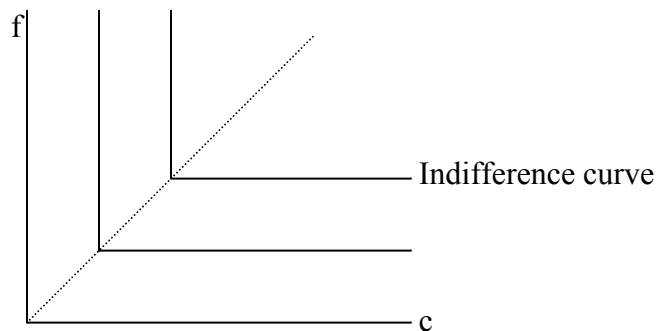
Problem 1.a



The opportunity cost of producing cars, i.e. the slope of the PPF, in Mars is smaller than that in Venus. Therefore Mars has comparative advantage in producing cars, and Venus has comparative advantage in producing food.

Problem 1.b

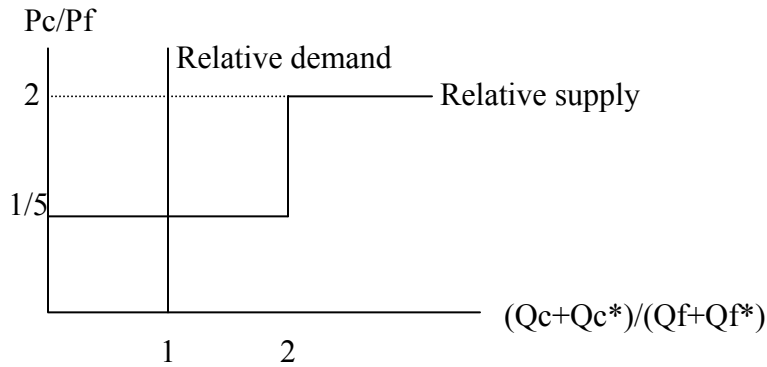
Under autarky, consumption point is the same as production point.  
The utility function is not differentiable; therefore we cannot find the first order condition. The utility function in fact implies two goods are perfect complements.



Mars – Optimal consumption choice:  $c = f$   
 Feasibility condition:  $10f + 2c = 120$   
 $\rightarrow c = f = 10$

Venus – Optimal consumption choice:  $c^* = f^*$   
 Feasibility condition:  $6c^* + 3f^* = 90$   
 $\rightarrow c^* = f^* = 10$

Problem 1.c



Given the optimal consumption choice in 1.b, relative demand curve is a vertical line. The relative demand and the relative supply intersects and the relative price is 1/5. Therefore the equilibrium relative price is 1/5. Given this price, Mars will produce both goods, and Venus will specialize in producing food.

Free trade equilibrium

We can normalize  $P_c$  as unity, then  $P_f=5$ .

Mars produces both goods. Venus specializes in food. Then  $Q_f^*=30$ .

Mars consumption choice:  $C_c = C_f = 10$ .

(This is the same as autarky consumption, because the relative price is the same.)

Venus consumption choice:  $C_c^*=C_f^*$

Mars budget constraint:  $10+5(10) = Q_c+5(Q_f)$

Venus budget constraint:  $C_c^*+5C_f^*=5(30)$

Car-market clearing condition:  $10+C_c^*=Q_c$

Food-market clearing condition:  $10+C_f^*=Q_f+30$

From the equations above,

$C_c=C_f=10$ .  $Q_c=35$ ,  $Q_f=5$ .

$C_c^*=C_f^*=25$ .  $Q_c^*=0$ ,  $Q_f^*=30$ .

Mars exports cars and imports food.

Cars exports =  $Q_c - C_c = 25$

Food imports =  $C_f - Q_f = 5$

Venus exports food and imports car.

Car imports =  $C_c^* - Q_c^* = 25$

Food exports =  $Q_f^* - C_f^* = 5$

Gains from trade

	Autarky	Free trade
Mars utility	10	10
Venus utility	10	25

It is evident that Venus gains from trade: Venus utility is higher under free trade.

Problem 1.d

Labor will move from the low-wage location to the high-wage location. With labor mobility, equilibrium wage must be equal in the two locations. Let  $W$  denote wage in Mars, and  $W^*$  denote wage in Venus.

$$W = Pc/2$$

$$W^* = Pf/3$$

The free trade relative price is  $1/5$ , and then free trade relative wage is  $3/10$ . As a result, labor will migrate from Mars to Venus. In the new equilibrium, the relative wage ratio must be unity. This implies the following.

$$(Pc/Pf)(3/2) = 1$$

$$Pc/Pf = 2/3$$

As long as specialization does not take place in both Mars and Venus, the relative price  $2/3$  cannot be realized. Thus migration must take place until each country produces according to comparative advantage, so that the relative supply curves depicted in 1.b overlaps with the relative demand curve. As a result, the relative production quantity in the migration equilibrium is 1.

$$Qc/Qf^* = 1$$

$$Qc = Qf^*$$

$$(120 - M)/2 = (90 + M)/3$$

$$M = 36$$

36 Martians will migrate to Venus. (Poor 84 Martians got stuck in their planet!)

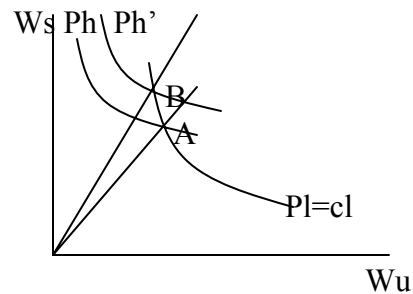
Problem 2.a

1. Stolper-Samuelson Result: If the relative price of an output rises, then the price of the factor used intensively in its production rises.
2. Factor Price Equalization (FPE): Free trade equalizes factor prices in both countries.
3. Heckscher-Ohlin Result: A country exports the commodity of which production intensively uses the abundant factor.
4. Rybczynski Theorem: When the quantity of a factor increases, the production in the sector that intensively uses that factor expands. At the same time, the production in the other sector contracts.

The four results can be categorized in the following way.

	Global	Local
Prices	FPE	Stolper-Samuelson
Quantities	Heckscher-Ohlin	Rybczynski

### Problem 2.b



Point A is the initial equilibrium. Suppose the price of high-tech good rises, while the price of low-tech good remains constant. Then the equilibrium moves to Point B. It is clear that the wage of skilled labor rises, but the wage of unskilled labor fall. In fact, the rise in the wage of skilled labor is higher than the rise in the price of high-tech good. This is the so-called magnification effect:

$$\text{change in } W_s > \text{change in } P_h > \text{change in } P_l > \text{change in } W_u$$

The magnification effect implies that the real reward of skilled labor rises, while that of unskilled labor fall, as a consequence of a rise in relative price of high-tech good.

When a country opened itself to free trade, the price of its export rises and the price of its imports fall. In other words, free trade raises the relative price of its export good. According to the Heckscher-Ohlin result, a country must export the good of which production requires an intensive use of the abundant factor. Therefore the Stolper-Samuelson result implies that the real reward of the abundant factor rises as a result of free trade. On the other hand, free trade reduces the real reward of the scarce factor.

### Problem 2.c

The violation of factor price equalization can be explained by two reasons.

- (1) The crucial assumption underlying the factor price equalization is the assumption that both countries have identical technology. This is not a realistic assumption.
- (2) Factor price equalization does not hold when endowment in each country is largely different.

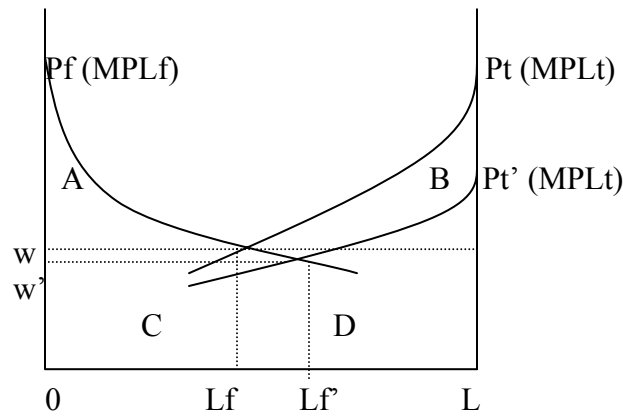
### Problem 2.d

STEP 1: Choose a group of countries, which most fit the assumptions of Heckscher-Ohlin model (i.e. identical preference, identical technology, different factor endowment). A group of OECD countries can be good candidates.

STEP 2: Look at data of (1) capital-labor abundance, or skilled-unskilled labor abundance; and (2) factor content in export and import goods of both countries.

STEP 3: Check if the capital-labor ratio in export and import goods matches with the capital-labor abundance. If it does match, then Heckscher-Ohlin prediction is valid.

Problem 3.a



- Pf = Price of fuzzy research
- Pt = Price of techy research
- Lf = Number of students working in fuzzy field
- Lt = Number of students working in techy field =  $L - L_f$
- Area A = Rent of fuzzy field professors
- Area B = Rent of techy field professors
- Area C = Rent of fuzzy field students
- Area D = Rent of techy field students

Problem 3.b

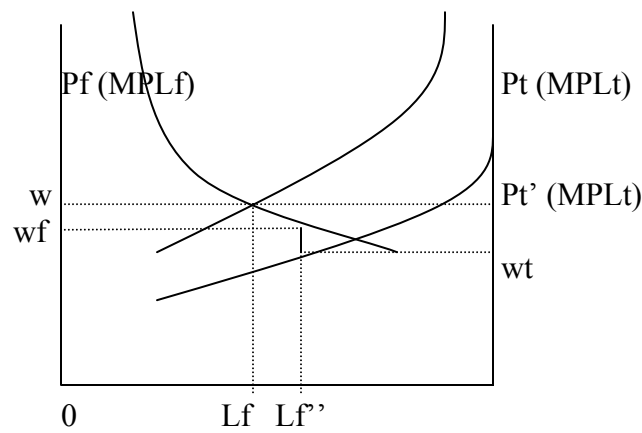
The drop of the price of techy research causes the marginal value of labor to shift downward to  $P_t'(MPL_t)$  (See the diagram in 3.a). The new marginal value of labor is not parallel to the initial one due to decreasing return in labor. As a result, students move to fuzzy field. The number of students moving to fuzzy field is  $L_f' - L_f$ .

The income distributional effects can be summarized as follows.

Profession	Effect of trade
Student	ambiguous
Fuzzy field professor	better off
Techy field professor	worse off

The fall in students wage is smaller than the fall in price of techy research, so their real wage in terms of techy research rises. In contrast, their real wage in terms of fuzzy research falls. Thus it is unambiguous whether students are worse off or better off. Professors actually receive the profits net of wage paid to students. Since the real wage of students in fuzzy field falls professors in fuzzy field are better off. In contrast, Professors in techy field are worse off with a similar reason.

Problem 3.c



Suppose switching sectors is costly. There are two possible outcomes.

First, if the switching cost is moderate, a number of students will move anyway. The number of students employed in fuzzy field becomes  $L_f''$ . The wages in the two sectors will become different. Let  $w_f$  be wage in fuzzy field, and  $w_t$  be wage in techy field. The following relationship holds in equilibrium.

- (1)  $w_f > w_t$
- (2)  $w_f - w_t = \text{switching cost}$

Second, if the cost is so large that there is no net wage gain, students will not move and there will be a wage gap between the two fields.

$$\text{techy field wage} = P_t'(MPL(L-L_f)) < w = \text{fuzzy field wage}$$

Problem 3.d

Anything goes!