Economics 165
Winter 2002
Problem Set #2

Problem 1: Consider the monopolistic competition model. Say we are looking at sailboat producers. Each producer has fixed costs of 10 million and marginal costs of 20,000 per boat. Each faces demand of the form $Q = S(1/n) - (P - P^a)/100{,}000)$ where S is the total quantity of boats sold. Suppose 2,500 sailboats are sold in the U.S. every year.

a) *Derive the marginal revenue of each producer (assuming that they are too small to affect the average price P^a). Set MR = MC as a profit maximizing monopolist does and use the symmetry assumption to derive the price P as a function of the number of firms n.*

To derive marginal revenue, start by expressing demand as an explicit function of price…

$$Q = S(1/n) - (P - P^a)/100{,}000)$$

$$P = 100{,}000/n - 100{,}000(Q/S) + P^a$$

Now multiply both sides by Q to find total revenue…

$$TR = PQ = (100{,}000/n)Q - 100{,}000(Q^2/S) + QP^a$$

Finally, differentiate with respect to Q to find marginal revenue…

$$MR = 100{,}000/n - 200{,}000(Q/S) + P^a$$

If we plug in 2,500 for S and set equal to MC = 20,000 we get…

$$\mathbf{MR = 100{,}000/n - 80Q + P^a} = 20{,}000 = MC$$

Solve for Q and you get…

$$Q = 1{,}250/n + P^a/80 - 250.$$

Using the symmetry assumption (all firms produce equal shares of the 2,500 boats) we get Q = 2,500/n, so. . .

$2{,}500/n = 1{,}250/n + P^a/80 - 250$. Also by symmetry (all firms charge the same price) $P = P^a$. Solve for P…

$$\mathbf{P = 100{,}000/n + 20{,}000}.$$

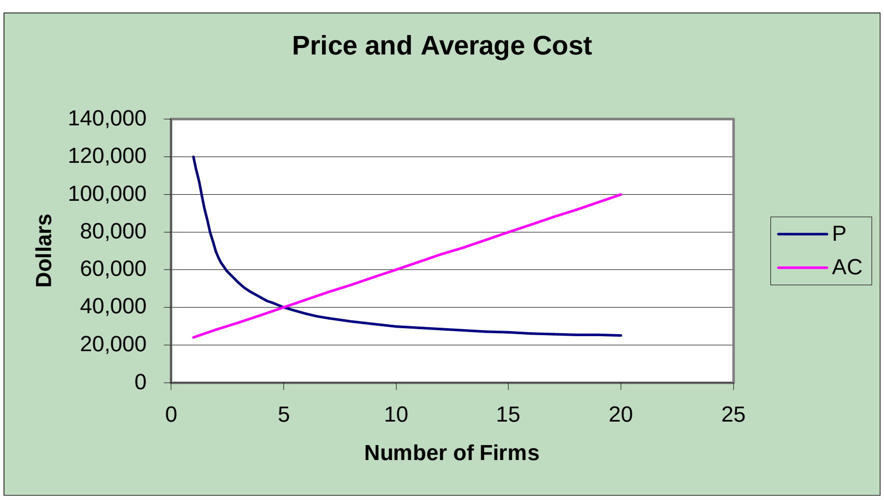
b) *In a diagram with P and average cost (AC) on the vertical axis and the number of firms on the horizontal axis, graph the relationship derived in a) as well as the average costs. Explain the intuition behind the slopes of these curves.*

To derive the average cost curve we first consider the total cost of production. We need to pay 10 million per firm (fixed costs) plus 20,000 per boat (variable costs). If we have n firms producing 2,500 boats then we have…

$$TC(n) = (10 \text{ million})n + (20{,}000)(2{,}500) = (10 \text{ million})n + (50 \text{ million}).$$

To find average cost simply divide total cost by Q (in this case 2,500)…

$$\mathbf{AC(n) = 4{,}000n + 20{,}000}.$$



The slope of the price function shows how prices fall as an additional firm enters the market. The slope of the average cost curve shows how costs rise as an additional firm enters the market.

c) *Why does their intersection represent the equilibrium? Calculate the equilibrium number of firms and the equilibrium price of a sailboat.*

The intersection represents an equilibrium because only at this point will profits be equal to zero. If we are at any point to the left, Price lies above Average Costs and so profits are positive. As a result, new firms will enter the market. If we are to the right, AC lies above P and profits are negative. As a result firms will exit. Only the case where **n = 5 and P = 40,000** represents an equilibrium in the long run.

d) Suppose there are six smaller countries with sales of 400 sailboats each. Calculate the equilibrium for one of them. Now, let the world trade and calculate the equilibrium for the U.S. trading with those six countries. Are there gains from trade?

Start with the equation for Marginal Revenue we found earlier…

$MR = 100{,}000/n – 200{,}000(Q/S) + P^a$

If we plug in 400 for S and set equal to MC = 20,000 we get…

$MR = 100{,}000/n – 500Q + P^a = 20{,}000 = MC$. Hence $Q = 200/n + P^a/500 – 40$. By symmetry $Q = 400/n$.

$400/n = 200/n + P/500 – 40$. Hence **P = 100,000/n + 20,000** just as before.

Total cost as a function of n is TC = (10 million)n + (400)(20,000) or **TC(n) = (10 million)n + 8 million.**

Hence **AC(n) = 25,000n + 20,000**.

Setting Price equal to Average Cost yields…

$100{,}000/n + 20{,}000 = 25{,}000n +20{,}000$.

So at equilibrium **n = 2 and P = 70,000**.

If we allow free trade amongst all countries, then we simply aggregate the seven autarkic markets into one big market. This market will consume 2,500 + 6(400) = 4,900 sailboats.

Once again, by calculating MR and setting equal to MC and assuming symmetry among the firms, we will find the price function is again **P = 100,000/n + 20,000**.

Total cost is now (10 million)n + 20,000(4,900). Hence **AC(n) = (10 million)n/4,900 + 20,000 = 2,041n + 20,000**.

Setting P = AC and solving for n yields…

n = 7 and P = 34,285.

There are definitely gains from trade. Consumers from all countries now pay a lower price for sailboats. The gain is especially large for consumers from the small countries. But what about producers? Are they worse off under free trade? The answer is no. In either case they were making zero economic profits, so they are indifferent. Finally, to the extent consumers like to have some choice in the sailboat they buy, they now can choose to purchase from one of 7 firms as opposed to only one of 5 or 2. Consumers are unambiguously better off and no one else loses.

Problem 2: Consider the market for steel. Home's (foreign's) inverse demand function for steel takes the form $p = 170 – Q^D$ ($p = 110 – Q^D$) and its inverse supply is $p = 70 + Q^S$ ($p = 10 + Q^S$).

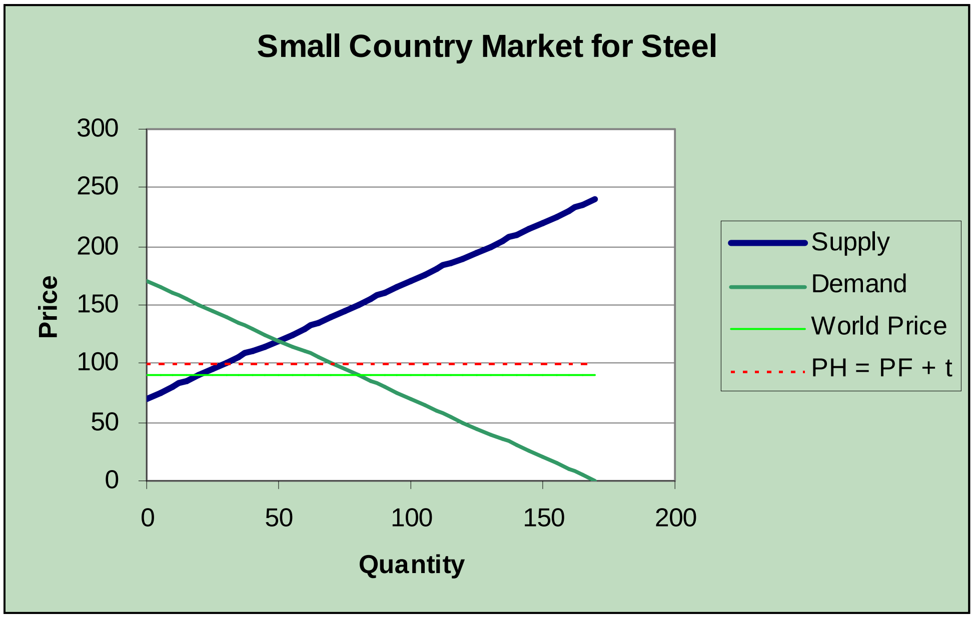
a) Derive the autarky equilibria for both countries, i.e. the equilibrium prices and quantities when they do not trade. Which country will export and which will import steel once trade is allowed.

Home: 170 – Q = 70 + Q. **Hence Q = 50; P = 120**.

Foreign: 110-Q = 10+Q. **Hence Q = 50; P = 60.**

Since Home has the higher autarky price it will import and Foreign will export once trade is allowed.

b) Suppose home is a small country that can import steel from the world market at a price of 90. Consider an import tariff of 10 per unit of steel. What will be the domestic price and the quantity imported? What will be the effects on producer profits, consumer surplus and government revenue? Does this policy raise national welfare?



The tariff of 10 will raise the domestic price from 90 to **100**. This in turn will decrease demand from 80 to 70 and increase supply from 20 to 30. (See domestic supply and demand functions.) As a result, imports (the difference between demand and supply, will shrink from 60 to **40**.

Prior to the tariff, price was 90, demand was 80 and domestic supply was 20. When this was the case…

CS = ½(80)(80) = 3,200

PS = ½(20)(20) = 200

G = 0

After the tariff, price is 100, demand is 70, domestic supply is 30 and imports are 40. Now…

CS = ½(70)(70) = 2,450

PS = ½(30)(30) = 450

G = (10)(40) = 400.

Hence consumer welfare was reduced by 750, producer welfare was increased by 250 and government revenues were increase by 400. In the aggregate, **national welfare was reduced by 100**.

c) *Repeat b) if home is not small but large and imports steel from the foreign country. Explain the concept of an optimal tariff. Find the optimal tariff.*

As a result of the tariff, the price in Home will be 10 higher than the price in Foreign. We need to find prices which satisfy $P^H = P^F + 10$, and for which Home's import demand is equal to Foreign's export supply. Express Home's Demand and Supply functions as $\mathbf{Q^D = 170 - P^H}$ and $\mathbf{Q^S = P^H - 70}$. If we subtract the quantity supplied from the quantity demanded, we'll get the quantity of imports. Hence…

$\mathbf{IM = 240\text{-}2P^H}$**.**

Foreign's Demand and Supply functions are $\mathbf{Q^D = 110 - P^F}$ and $\mathbf{Q^S = P^F - 10}$. If we subtract the quantity demanded from the quantity supplied, we'll get the quantity of exports. Hence…

$\mathbf{X = 2P^F - 120}$. Set X = IM and substitute $P^H = P^F + 10$ to get…

$240 - 2P^F - 20 = 2P^F - 120$. Hence $\mathbf{P^F = 85}$, so $\mathbf{P^H = 95}$.

Now in the home country after the tariff, price is 95, demand is 75, domestic supply is 25 and imports are 50.

Hence…

$CS = ½(75)(75) = 2,812.5$

$PS = ½(25)(25) = 312.5$

$G = (10)(50) = 500$

Comparing these values to free trade, we see that consumer welfare is reduced by 387.5, producer welfare is increased by 112.5 and government revenues increase by 500. In this case, **national welfare is increased by 225**.

We can see that in the case of the large country a tariff can improve national welfare. This raises the question of what level of tariff will maximize national welfare. This would be the optimal tariff. Such a tariff should maximize the sum of CS + PS + G. In order to calculate such a tariff, we must first express all three of these expressions in terms of t.

Recall the import demand and export supply equations…

$IM = 240-2P^H$.

$X = 2P^F – 120$. Set X = IM and substitute $P^H = P^F + t$ to get…

$240 – 2P^F – 2t = 2P^F – 120$. Hence **$P^F = 90 – t/2$**, so **$P^H = 90 + t/2$**

So in the Home country price equals 90 + t/2, which means that consumers buy 80 – t/2, domestic producers make 20 + t/2 and the country imports 60 – t.

$CS = ½(80 – t/2)^2$

$PS = ½(20 + t/2)^2$

$G = t(60-t)$. If we add these up, differentiate and set equal to zero, we get…

$-1/2(80-t/2) + 1/2(20 + t/2) + 60 – 2t = 0$. $30 = (3/2)t$. Hence the optimal tariff is **$t = 20$.**

d) *Consider an export subsidy of 10 per unit of steel in the foreign country. Analyze the effects of this subsidy for the large country case and contrast them with the effects of an export tax. Do you see any possibility of an optimal export subsidy or perhaps an optimal export tax?*

Once again start with the import demand and export supply equations…

$IM = 240-2P^H$.

$X = 2P^F – 120$. Set X = IM. This time substitute $P^H = P^F – s$, to get…

$240 – 2P^F + 2s = 2P^F – 120$. Hence **$P^F = 90 + s/2$**, and **$P^H = 90 – s/2$**.

If the export subsidy is 10, then the price in foreign is 95, domestic demand is 15, supply is 85, exports are 70. In the absence of a subsidy (free trade) the price is 90, demand is 20, supply is 80 exports are 60.

Under free trade, in foreign…

$CS = ½(20)(20) = 200$

$PS = ½(80)(80) = 3,200$

$G = 0$

With an export subsidy of 10…

$CS = ½(15)(15) = 112.5$

$PS = ½(85)(85) = 3,612.5$

$G = -10(70) = -700$

In this case, consumers lose 87.5, producers gain 412.5 and the government loses 700. **The total loss is 375**.

To find the optimal export subsidy recall that $P^F = 90 + s/2$ and $P^H = 90 – s/2$. In this case foreign's demand is 20 – s/2, supply is 80 + s/2, exports = 60 + s. Hence…

$CS = ½(20 – s/2)^2$

$PS = ½(80 + s/2)^2$

$G = -s(60 + s)$. Add up and differentiate with respect to s and set equal to zero…

$-1/2(20 – s/2) + 1/2(80 + s/2) – 60 –2s = 0$. Hence, $30 = -(3/2)s$. Or **s = -20**. This implies that the optimal policy for foreign will be an **export tax of 20**.