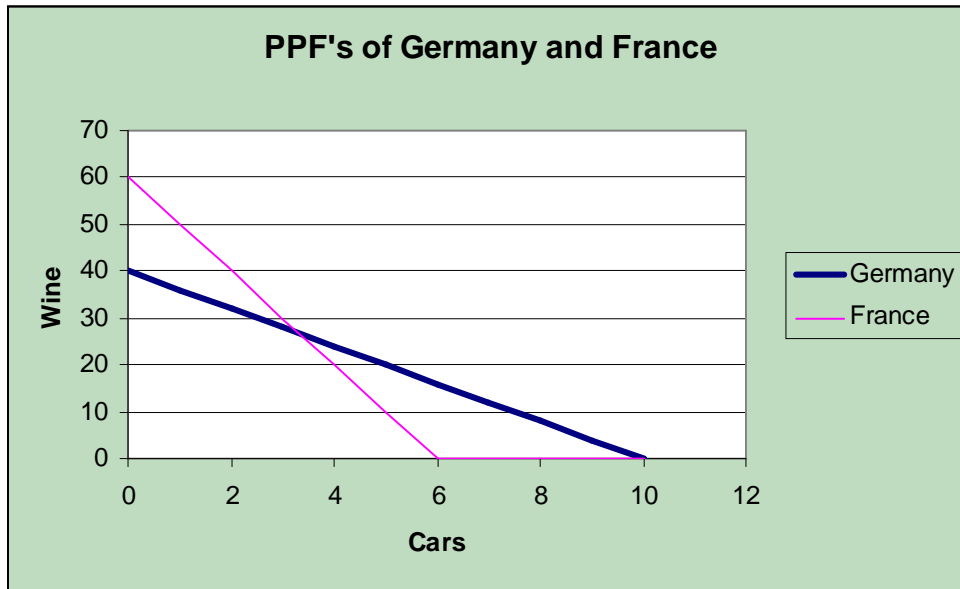


Problem 1: Let Germany and France have respective labor forces of 80 and 60. Suppose both countries produce wine and cars according to the following unit labor requirements: $a_w^G = 2$; $a_c^G = 8$; $a_w^F = 1$; $a_c^F = 10$. Further, suppose that the representative German and Frenchman both have utility given by $U^i(Q_w, Q_c) = Q_w Q_c$.

- a) Depict the PPF for both countries and label the intercepts as well as the slope (include their exact values, pls). Which country has the comparative advantage in producing wine/cars?

If we plot cars on the x-axis and wine on the y, we see that the slope of the German PPF is -4 and that of the French is -10 . Since the opportunity cost of a car is 4 bottles of wine in Germany and 10 bottles in France, we see that the Germans have a comparative advantage in car production and the French have a comparative advantage in the production of wine.



- b) Suppose each country wanted to go it alone. What would be the quantities produced (=consumed) and the utility levels obtained under autarky? (A little help: recall from micro that in the consumption optimum $MRS = MU_c/MU_w = p_c/p_w$)

For each country, $MRS = MU_c/MU_w = W/C$. The price ratio will be the slope of the PPF in each country. In Germany this is 4. Hence, $W = 4C$. The point on Germany's PPF where $W = 4C$ is when $C = 5$ and $W = 20$. Hence, this is what Germany produces. Since Germany has a population of 80, each consumer will get $1/16$ units of car and $1/4$ of a unit of wine. Hence, the **utility of the representative German is $1/64$** .

In France, the price ratio is 10, so $W = 10C$. The only point on the French PPF satisfying this condition is at $C = 3$ and $W = 30$. This is France's production choice. Since the population is 60, each worker will consume $1/20$ cars and $1/2$ wine. Consequently, the **utility of the representative Frenchman is $1/40$** .

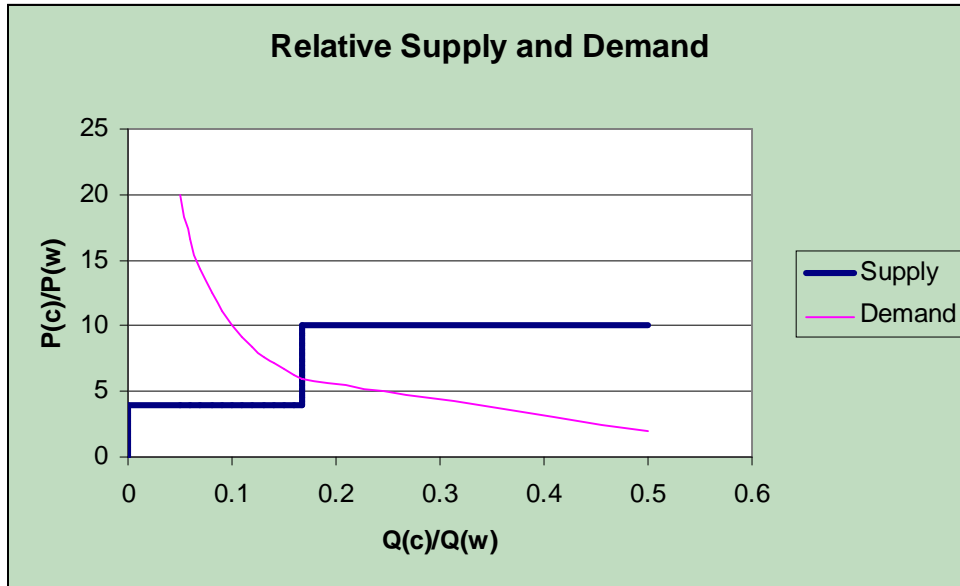
- c) Now consider free trade. Depict the world's relative supply function (again, pls label and provide numbers). What is the world's free trade equilibrium relative price? What quantities does each country produce, how much do they import/export? What are the utility levels obtained? How high are their respective wages?

Consider what each country will do under free trade. Given a set of prices, each country will want to produce at the point on the PPF that will generate the most wealth. Then given those same prices and that level of wealth, each will chose to consume at the point that maximizes utility.

Start with Germany. If the price of cars is very low relative to the price of wine, Germany will want to produce only wine. For example, if $p_c = 2$ and $p_w = 1$, then if Germany produces only wine, it will have a GDP of $40(1) = 40$ and if it produces only cars its GDP will be $10(2) = 20$. As a result, Germany will produce only wine. Since you always need to reduce wine production by 4 units if you increase car production by 1 unit, Germany will never produce cars as long as $p_c/p_w < 4$. At the point where $p_c/p_w = 4$, Germany will be indifferent to producing cars or wine. That is,

any point on the PPF will generate the same GDP for the country. This corresponds to the flat part of the relative supply graph. Finally if $p_c/p_w > 4$, Germany will produce only cars.

France goes through a similar though process, but since the slope of its PPF is -10, the relative price at which France is indifferent between producing cars and wine is $p_c/p_w = 10$. For a lower price ratio, France will specialize completely in wine production and for a higher ratio it will specialize completely in car production.



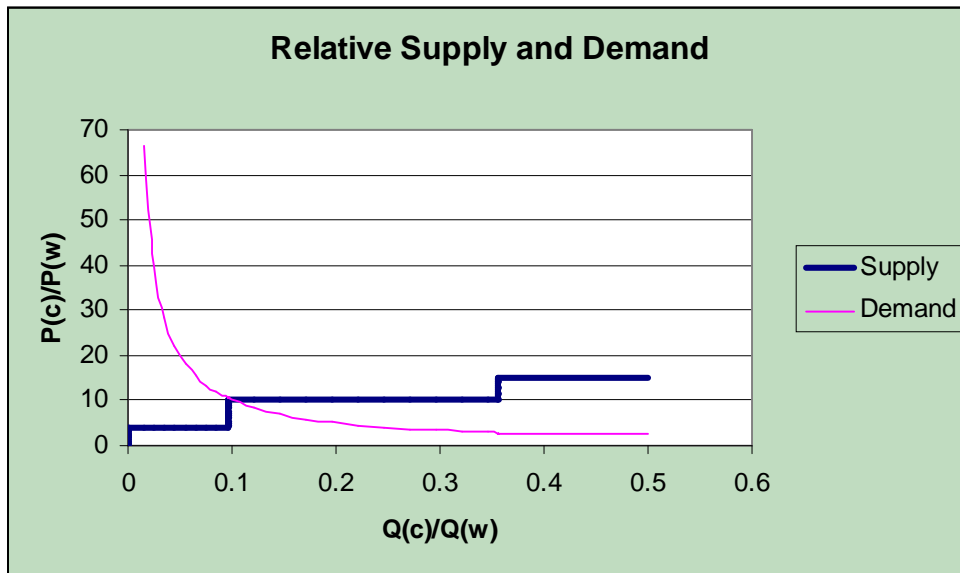
The relative demand curve can be found by aggregating consumer demand. This task is greatly simplified since all consumers have the same utility function. Since the utility function is Cobb-Douglas, we know that all consumers will have constant expenditure shares. (If you don't recall this fact about Cobb-Douglas utility, just solve the consumer's utility max problem.) This means that every individual has demand functions given by $W = M/2p_w$; and $C = M/2p_c$, where M is wealth and the 2 in the denominator comes from the result that all workers spend half their income on C and half on W . If we divide C by W we find that the relative demand function is: $Q_c/Q_w = p_w/p_c$. The only point on the supply curve that satisfies this equation is where $Q_c/Q_w = 1/6$ and $p_c/p_w = 6$. This is where the market will be in equilibrium.

When $p_c/p_w = 6$, we know that **Germany will produce only cars (10 of them) and France will produce only wine (60 units)**. The value of Germany's 10 cars is 60 and it will spend half its wealth on cars and half on wine (because of the representative agent's utility function). Hence **Germany will consume $30/6 = 5$ cars and $30/1 = 30$ units of wine**. France's 60 bottles of wine are worth 60. Since France has the same preferences as Germany and the same wealth, it follows that facing the same prices (one world price) France will consume the same bundle. Hence 5 cars and 30 units of wine. **This means that Germany exports 5 cars and imports 30 units of wine, while the opposite is true in France.** Because Germany has a population of 80, each person there consumes $1/16$ units of cars and $3/8$ units of wine. In France (population 60) each person consumes $1/12$ units of cars and $1/2$ units of wine. Since Germans consume $3/4$ the amount that French people consume, their wages are $3/4$ as high. That is $w_G = (3/4)w_F$. Finally, plugging the consumption amount into the utility functions reveals that the **representative consumer in Germany receives utility of $(1/16)(3/8) = 3/128$, and the representative consumer in France receives a utility of $(1/12)(1/2) = 1/24$.**

- d) Consider a third country: Spain, which has a labor force of 45 and unit labor requirements $a_w^S = 1$ and $a_c^S = 15$. Repeat c) for all three countries. Does Spain gain from trade? What about the other two?

Start by noting that the Spanish PPF will have a slope of -15. This means that for them in autarky, $p_c/p_w = 15$. At these prices, consumers will maximize utility by consuming in the aggregate where $W/C = 15$, or $W = 15C$. The only point on Spain's PPF satisfying this condition is $W = 22.5$ and $C = 1.5$. At this level of production each of the 45 workers will receive $1/2$ units of wine and $1/30$ units of cars. This generates a utility of $1/60$ for the consumers.

Since Spanish people have the same utility function as the French and Germans, the relative demand curve is unchanged. However, the relative supply curve will change to reflect the addition of Spain's production. We see from part c) that there are plateaus on the supply curve when the relative price is 4 and 10. These represent the prices at which Germany and France, respectively, are indifferent to where they produce on their PPFs. Spain is indifferent at a price ratio of 15, so now we add a new plateau. The resulting graph is...



The price ratio that clears the world market is $p_c/p_w = 10$. (We will also see this when we solve for the allocation.) At this price ratio, **Germany specializes in car production (producing 10 cars), Spain specializes in wine production (producing 45 units of wine)** and France is indifferent amongst all the production points on its PPF (all will give France the same GDP). When Germany produces 10 cars, its GDP is $10(10) = 100$. When Spain produces 45 units of wine, its GDP is $45(1) = 45$. Regardless of where France produces on its PPF, it will have GDP = 60. (It could produce 60 units of wine worth 60, 10 units of cars worth 60 or any combination in between, but regardless, the value of Frances production will be 60.)

When we know the wealth of every country, we know what it will consume. Once again, recall that due to the utility function, each consumer will spend half his wealth on cars and half on wine. **So the Germans will be able to afford $50/10 = 5$ cars and $50/1 = 50$ units of wine. The French will buy $30/10 = 3$ cars and $30/1 = 30$ units of wine and finally the Spanish will consume $22.5/10 = 2.25$ cars and $22.5/1 = 22.5$ units of wine.** If we add up all the consumptions of cars, we see that aggregate car consumption is $C = 5 + 3 + 2.25 = 10.25$; and aggregate wine production is $W = 50 + 30 + 22.5 = 102.5$. Since Germany is supplying 10 cars and aggregate car demand is 10.25, it must be the case that **France produces .25 cars**. Since Spain supplies 45 units of wine and 102.5 are consumed in the aggregate, **France is producing 57.5 units of wine**. Sure enough, this production point is on the French PPF, so the relative price of 10 is supportable in equilibrium.

As a result, **Germany exports 5 cars and imports 50 wine, Spain exports 22.5 wine and imports 2.25 cars, and France exports 27.5 wine and imports 2.75 cars.** When we divide country consumption by population we find that a **German worker consumes $1/16$ cars and $5/8$ wine; a French worker consumes $1/20$ cars and $1/2$ wine; and a Spaniard will consume $(2.25)/45 = 1/20$ cars and $1/2$ wine.** Since the Frenchman and the Spaniard consume the same bundle, they must have the same wage. The German consumes 25% more. Hence $w_F = w_S = (.8)w_G$. Plugging consumption bundles into the utility function reveals that $U_F = U_S = 1/40$ and $U_G = 5/128$. Comparing these values to autarky, we see that **Germany and Spain gain from trade and France neither gains nor loses.**

Problem 2:

a) *List the four main results of the Heckscher-Ohlin model and state each in your own words..*

- i) **Heckscher-Ohlin Result:** If two countries differ only in their relative factor endowments and they are opened up to trade freely, each country will export the good whose production process uses relatively more intensively that factor in which the country is relatively more abundant.
- ii) **Rybczynski Effect:** If a country experiences an increase in its endowment of one of the factors, production of the good which utilizes that factor relatively more intensively will increase more than proportionately to the increase in the factor endowment, while production of the other good will actually decrease.
- iii) **Stolper-Samuelson Effect:** If the market price of one of the goods increases, the return to the factor utilized relatively more intensively in the production of that good will increase more than proportionately to the increase in the output price.

- iv) **Factor-Price Equalization:** Under trade in the H-O model, as long as both countries are producing both goods, trade in outputs is fully proxy for trade in factors. That is, the return to factors will be equalized across countries.

b) *Derive the H-O result graphically. Provide two real world examples of this result (if you cannot find any, feel free to provide counter examples).*

For graphs of the H-O result see the handout entitled *Notes on HO – part 2* pages 2 and 3.

Some examples of the H-O result...

The U.S. is very richly endowed with arable land. Saudi Arabia has generous deposits of oil. Consequently, the U.S. exports food and Saudi Arabia exports oil.

Canada has a lot of trees. France has a good deal of land and other resources suitable to making wine. Canada exports lumber while France exports wine.

A couple places where H-O doesn't work terribly well...

The U.S. has a relatively high capital/labor ratio. Hence under the H-O model, we'd expect the U.S. to import relatively labor intensive goods and export relatively capital intensive goods. Turns out that the opposite is true when you look at data.

The U.S. is relatively well supplied with skilled labor. Under free trade you'd expect the U.S. to export skilled labor intensive goods and import unskilled labor intensive goods. This is in fact the case. You'd also expect trade to increase the return to skilled labor while decreasing the return to unskilled labor, resulting in increasing income inequality. This also, appears to be the case. However, you'd expect the opposite to be true in our trading partners (i.e. income inequality in poorer countries should be decreased by trade). This is not supported by the data.

c) *In your diagram(s) from b) describe what happens once one country becomes completely specialized. Which of the four results from a) no longer holds in this case?*

If one country completely specializes, then graphically, it produces on one on the corners of its PPF (in the graph with the PPFs) or on one of the borders of the parallelogram (in the graph with the parallelogram). Factor price equalization need no longer hold in this case.

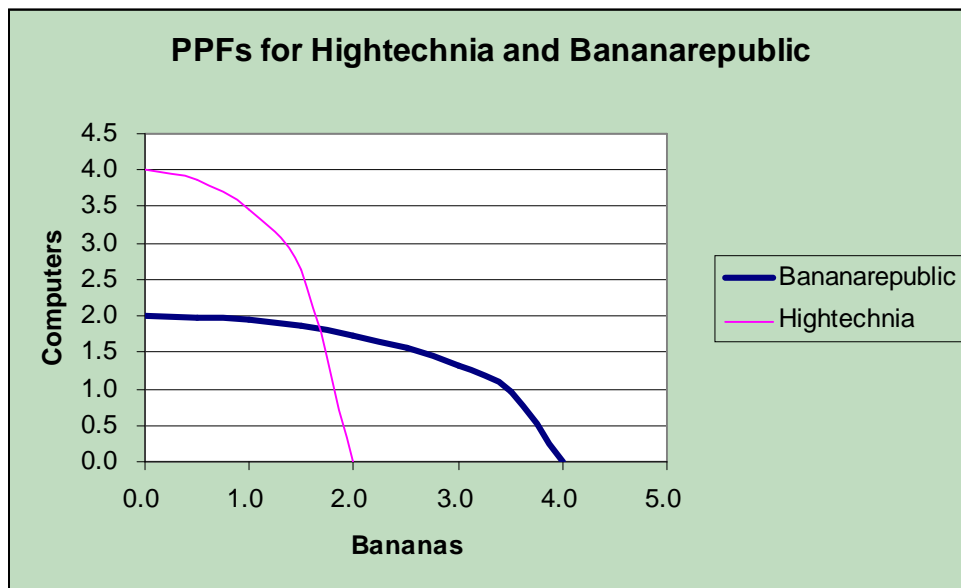
d) *Show the gains from trade in the context of the H-O model. How do you reconcile this finding with Stolper-Samuelson's result that one factor gains but the other loses?*

If you look at the graph with the two PPFs you'll notice that under trade both countries choose to consume at a point above their PPF. They must be better off since they could not choose this consumption bundle under autarky (it is revealed preferred). This is still consistent with the SS result that one factor gains and the other loses. Underlying the aggregate gains to free trade is a redistributive conflict. The loss to the import competing factor is smaller in magnitude than the gain to the factor that is being exported by proxy. Hence, it should be possible to tax away some of the gains of the winning factor in order to completely compensate the losses of the losing factor and still have surplus left over.

Problem 3: The two countries Hightechnia and Bananarepublic produce computers C and bananas B. Unskilled labor U is mobile between both sectors whereas skilled labor S is specific to computers and land L to bananas. Hightechnia is abundant in skilled labor while Bananarepublic is abundant in land. Both countries have identical (and homothetic) preferences. Initially they do not trade.

a) *Derive each country's PPF, emphasizing their difference. How will their autarky price ratios p_C/p_B differ? In which direction will these price ratios change as they move from autarky to free trade?*

Because of the relative factor endowments, when we graph the PPFs we notice that Hightechnia can produce more computers and Bananarepublic can produce more bananas. We would expect p_C/p_B to be smaller in Hightechnia than in Bananarepublic in autarky because it has a comparative advantage in computer production. When free trade is opened up, the price ratio will rise in Hightechnia and it will fall in Bananarepublic. Hence, the free trade price ratio will lie somewhere between the autarkic price ratios.



- b) *How does the move to free trade affect the real incomes in terms of bananas and computers of U, S, and L in Hightechnia? What can you say about consumption possibilities of the country as a whole?*

We know that when we move to free trade the price of computers in Hightechnia will increase and the price of bananas will decrease. Without loss of generality, we can renormalize prices in such a way that the price of computers increases while the price of bananas remains constant. When the price of computers increases, the nominal wages of all factors involved in computer production will initially increase proportionately. But this would mean that Unskilled workers in the computer industry earned more than in the banana industry. As a result more labor will be attracted to the computer industry and wages in the computer industry would fall part of the way back down while those in the banana industry rose. In the end, the wages of all unskilled laborer will have risen less than proportionately to the increase in computer prices.

The reduction of wages in the computer industry resulting from the movement in U from B to C will further increase wages for S. Since S has seen its return increase more than proportionately to p_C , the purchasing power of S over C has increased. Also since the price of bananas is unchanged S has increased purchasing power over B as well. Hence the real income to S has been unambiguously increased.

We saw earlier that nominal wages to U rise less than proportionately to the price increase of C. Hence real wages have fallen in terms of C (unskilled workers can't afford as much in computers as they could in autarky). However, since nominal wages rose and the price of bananas is unchanged, real wages have risen in terms of bananas. The total effect on U is ambiguous. If the representative consumption basket is weighted heavily toward computers then U is worse off under trade. If consumption is weighted toward bananas, U is better off.

Nominal returns to L have fallen since the price of bananas is unchanged but the wage to labor has increased. This will unambiguously decrease the real wage of L.

The consumption possibilities of the country as a whole have increased, since the country will now be consuming at a point above the PPF.

- c) *Repeat b) for Bananarepublic.*

The story here will be just the opposite. Now the price of bananas will rise and the price of computers will fall. Without loss of generality, we will renormalize prices so that only the price of bananas changes (it increases). Here real wages for L will be unambiguously increased, real wages for S will be unambiguously decreased and there will be an ambiguous outcome for U. If U's consumption is weighted toward computers, U will be better off, and if consumption is weighted toward bananas U will be worse off.

This country too benefits in the aggregate as consumption will lie above the PPF.