

INTERNATIONAL ECONOMICS
HOMEWORK 3
22/11/2013

Consider an economy characterised by the following utility function: $U = \sum_{i=1}^n X_i^\beta$. Where X_i is the consumption of good i , $0 < \beta < 1$ is a constant and n is an endogenously determined number of produced in the economy. Each good that is produced is subject to diminishing marginal utility, therefore the consumer would always rather have one unit each of two goods, say X_1 and X_2 , rather than two units of either. There is only one factor of production labour, and the total labour endowment is \bar{L} . Assume that labor is the numeraire.

- a. Set up the consumer's utility maximization exercise, and use the first order conditions to show

that the demand for good i is given by: $X_i = \frac{\bar{L}}{p_{x_i}^\sigma P}$,

where $\sigma = \frac{1}{1-\alpha}$ and $P = \sum_{j=1}^n p_{x_j}^{-\alpha\sigma}$.

- b. Assuming that P is constant (i.e. there are many X producers and each producer treats P as being unaffected by its decision to choose a price), show that the elasticity of demand for good i is given by:

$$e_i = -\frac{dX_i}{dp_{x_i}} \cdot \frac{p_{x_i}}{X_i} = \sigma = \frac{1}{1-\alpha}$$

- c. Suppose production of each good i has the following total cost function:
 $TC_{x_i} = F + MC_{x_i} \cdot X_i$

where F is fixed cost and MC_{x_i} is the constant marginal cost of producing good i and X_i is the output of good i (which by market clearing for good i is equal to the demand for good i). Set up the firm's (producing good i) profit maximization exercise and use the first order condition to obtain an expression for the price of good i . (Hint - first obtain the expression for total revenue and then using expressions for total revenue and total cost obtain the expression for profits. The firm chooses price and not output.)

- d. Assuming free entry and exit, profit for the firm is driven to zero, i.e. price is equal to average cost. Use the expression for price obtained in previous part to obtain the following expression

for the output (also demand): $X_i = \frac{\alpha F}{MC_{x_i} (1-\alpha)}$

- e. Use the expression for X_i obtained in the previous part to obtain an expression for TC_{x_i} . Show that the total cost is the same for every good i . The total cost of producing n goods must be equal to the total value of labor in the economy ($w\bar{L} = \bar{L}$). Use this to obtain an expression for the number of goods, n .

- f. What happens to number of goods produced if labour endowment was to double? Based on your answer what can you say about the effect of trade between two such economies if they open up to trade?